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THESIS

EVALUATION OF A MODIFIED AMSAA CONTINUOUS
RELIABILITY GROWTH MODEL USING
FAILURE DISCOUNTING AND WEIGHTING FACTORS

by

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Failure discounting is the practice of removing fractions of failures from test data after corrective actions have been taken and no failures due to the same cause have reoccurred. This thesis examines the effect of discounting failures and weighting test data on the accuracy of an existing reliability growth model, labeled the Modified AMSAA model. Computer simulation is used to evaluate the mean and mean square error of failure rate estimates under the model for a variety of reliability growth patterns each with several discounting and weighting scenarios. Exponential failure times are assumed and testing is truncated at two failures in each test phase. Failure discounting tended to decrease the mean square error slightly for growth patterns with a continual drop in failure rate for each new test phase, but tended to increase the mean square error for other patterns. The Modified AMSAA model is also shown to be superior to the standard AMSAA reliability growth model in bias and mean square error. No discernable benefits due to weighting the data were detected.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, with time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user

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I. BACKGROUND

The AMSAA model is a widely used cumulative reliability growth model. It was developed by the Army Material Systems Analysis Activity (AMSAA) at Aberdeen Proving Grounds, Maryland. Failure discounting applied to this and other reliability growth models is becoming popular. In previous work by Woods [ref. 1] the Modified AMSAA model appears to provide more accurate estimates of current reliability for various patterns of reliability growth than does the standard AMSAA model.

The primary purpose of this thesis is to determine the effect of failure discounting and data weighting on the Modified AMSAA model. Secondary purposes are to compare the accuracies of the AMSAA model and the Modified AMSAA models. These comparisons are made for a variety of reliability growth patterns, failure discounting rates, and weighting scenarios.

Throughout this thesis the phrase 'test phase' refers to a collection of tests wherein the nontruncated failure times of all items tested are independent and exponentially distributed with a common failure rate. Within each phase, n items are tested until r fail, and testing is performed sequentially by phase. Testing in phase i+1 is initiated after the failures in phase i have been analyzed and appropriate changes have been made. Both n and r are input parameters to the simulation program used to generate test data.

In application of the Modified AMSAA model, if the data has a Weibull distribution with shape parameter, β, then raising the data to the β power yields exponential data. Applying the model to this transformed data should yield estimates as accurate as those indicated in this thesis.

II. THE AMSAA RELIABILITY GROWTH MODEL

The AMSAA reliability growth model has its roots in a report by J. T. Duane of General Electric Company published in 1962. In this report he presented observations on failure data for five types of systems during their development programs at General Electric. His analysis revealed that the observed cumulative failure rate versus the cumulative operating hours fell close to a straight line when plotted on log-log paper. That is, if N(t) denotes the total number of failures observed in t hours of testing and C(t) is the cumulative failure rate observed over that same time, then log C(t) is almost a linear function of log t [ref. 2: p.29]. The model which Duane used to interpret those plots has a cumulative failure rate estimate C(t) with expectation given by

$$E[C(t)] = E[N(t)/t] = \lambda t^{\beta-1} \qquad (2.1).$$

Letting $\lambda = b$ and $\beta = 1-a$ results in

$$E[C(t)] = bt^{-a} \qquad (2.2).$$

Duane defined the instantaneous failure rate r(t) by d/(E[N(t)])/dt. Applying this to $E[N(t)] = \lambda t^{\beta}$ and differentiating yields

$$r(t) = \lambda\beta t^{\beta-1} = (1-a)bt^{-a} \qquad (2.3),$$

where t denotes total accumulated test time [ref. 2: p.29].

The expression $\lambda\beta t^{\beta-1}$ is the failure rate function for the Weibull distribution. This does not mean, however, that the failure times have a Weibull distribution, because the underlying distribution of the failure times changes each time a change is made in the hardware. It only means that the failure rate estimate r(t), as a function of the cumulative test time, t, is changing in accordance with the expression in in equation (2.3).

This model mixes data from different populations to obtain an estimate of current reliability. This apparent weakness motivated the development of the Modified AMSAA model. The unmodified AMSAA model is also referred to as the cumulative AMSAA model in some graphs that follow.

"The AMSAA Reliability Growth Model assumes that system failures during a development testing phase follow the (nonhomogeneous) Poisson Process with Weibull intensity $r(t) = \lambda\beta t^{\beta-1}$, where $\lambda > 0$, $\beta > 0$." [ref. 2: p. 29] The parameters are estimated in the following manner where N = the number of failures and X_i = total test time up to i^{th} failure.

$$r(t) = \lambda\beta t^{\beta-1} = (1-a)bt^{-a} \tag{2.4}$$

$$1-\hat{a} = \hat{\beta} = \frac{N}{\sum_{i=1}^{N-1} \log(X_N/X_i)} \tag{2.5}$$

$$\hat{b} = \hat{\lambda} = \frac{N}{(X_N)^{\hat{\beta}}} \tag{2.6}$$

$$\hat{r}(t) = \hat{\lambda}\hat{\beta}t^{\hat{\beta}-1} = (1-\hat{a})\hat{b}t^{-\hat{a}} \tag{2.7}.$$

Note that:

$$\hat{r}(T) = \frac{N}{T^{\hat{\beta}}}\hat{\beta}T^{\hat{\beta}-1} = \frac{\hat{\beta}N}{T} \tag{2.8},$$

which "...is equivalent to using the exponential method but purging

$(1-\hat{\beta})N$ failures" [ref. 2: p.30]. This indicates that the model is self purging, and no further failure removal method is required. Some people have developed failure discounting methods for this model.

III. THE MODIFIED AMSAA MODEL

The Modified AMSAA Model uses exponential regression to compute estimates for a and b after each phase. Test time within each phase is treated as an independent observation, and the estimator for failure rate within each phase, given by equation (3.1), is used as the dependent variable in the regression analysis. The cumulative AMSAA model requires one estimate of a and b to cover all phases.

The Modified AMSAA model uses the same expression for current failure rate as given in the AMSAA model, namely, $\lambda_{TTj} = (1-a)b(TT_j)^{-a}$, where TT_j is the total accumulated test time over all phases at the end of phase j [ref. 1: p. 3-1]. Within each test phase j, a failure rate estimate is computed by

$$\hat{\lambda}_j = \frac{2F_j - 1}{2F_j} \bullet \frac{F_j}{T_j} \qquad \text{if } F_j > 1, \quad \text{or}$$

$$\hat{\lambda}_j = \frac{0.5}{T_j} \qquad \text{if } F_j \leq 1. \qquad (3.1)$$

where F_j is the number of observed failures in phase j, and T_j is the total accumulated test time in phase j. Explanation for the bias correction, $(2F_j-1)/2F_j$, is developed in Appendix A.

Let j denote the current phase of testing just completed, and let

$TT_i = \sum_{k=1}^{i} T_k$. $Y_i = \ln \hat{\lambda}_i$, $X_i = \ln TT_i$, for i = 1,2,...,j. Let

$\bar{Y}_j = (Y_1+Y_2+\ldots+Y_j)/j$ and $\bar{X}_j = (X_1+X_2+\ldots+X_j)/j$. The current failure rate estimate, $\hat{\lambda}_{TTj}$ is given by

$$\hat{\lambda}_{TTj} = (1-\hat{a})\hat{b}(TT_j)^{-\hat{a}} \qquad (3.2).$$

Equations for $\hat{a}_j$ and $\hat{b}_j$ are given by

$$\hat{a}_j = \frac{\sum_{i=1}^{j} X_i Y_i - \bar{Y}_j \sum_{i=1}^{j} X_i}{\bar{X}_j \sum_{i=1}^{j} X_i - \sum_{i=1}^{j} X_i^2} \qquad (3.3)$$

$$\hat{b}_j = \frac{1}{1-\hat{a}_j} \exp(\bar{Y}_j + \hat{a}_j \bar{X}_j) \qquad (3.4)$$

for $j = 2,3,\ldots$. The regression requires observations from at least two test phases. The instantaneous failure rate estimate given by equation (3.1) for j=1 is used for the first phase [ref. 1: p. 3-2]. Equations (3.3) and (3.4) are developed in appendix A.

A. WEIGHTING

This model can be modified further by the application of regression weights in an attempt to make it more sensitive to changes in failure rate between phases. Weighted regression is generally used when some observations are less reliable than others. Generally this implies unequal variance among the observations [ref. 3: p. 77]. To accommodate this phenomena, weights used in weighted regression are heavier for phases with lower variance or more reliable observations. We can use a weighting scheme based on estimated variance or just use a set of weights chosen by some other scheme.

The principle of weighted least squares regression is to minimize the weighted sum of squared differences between observed values and predicted values, $\Sigma\, w_i\, (Y_i - \alpha - \beta\, X_i)^2$. In weighted regression the normal least squares regression equations become

$$\hat{\beta} = \frac{\sum_{i=1}^{N} (X_i - \overline{X}_w)\; Y_i w_i}{\sum_{i=1}^{N} (X_i - \overline{X}_w)^2\; w_i} \qquad (3.5) \quad \text{and}$$

$$\hat{\alpha} = \overline{Y}_w - \beta\, \overline{X}_w \qquad (3.6)$$

where $\overline{X}_w$ and $\overline{Y}_w$ are the weighted averages of the data points, and the linear relationship between X_i and Y_i is described by the model $Y_i = \alpha + \beta\, X_i$ [ref. 4: p. 89].

In this thesis, two weighting procedures are analyzed. In method one, the weights are calculated within the program. In this case the quantities $w_i^* = f^{1/i}$ are computed where f is a number between 0 and 1 chosen by the user as a parameter for the program. These quantities are normalized to yield the weights. With this procedure, the data from the current phase is always given the most weight when the instantaneous failure rate is calculated for that phase. The smaller the value of f, the greater the effect of weighting.

When the user chooses the weights (method two), the chosen values for each phase, w_i^*, are read in from a data file. In both methods the w_i^* values are normalized to obtain weights, w_i, to use in the regression equations. The resulting weights are given by

$$w_i = \frac{w_i^*}{\sum_{k=1}^{j} w_k^*}, \quad i = 1,2,\ldots,j \qquad (3.7).$$

When using regression weights with the modified AMSAA model, the data pairs ($\ln \hat{\lambda}_i$, $\ln TT_i$) are used in the weighted regression equations as follows: $Y_i = \ln \hat{\lambda}_i$, $X_i = \ln TT_i$,

$\overline{Y}_{wj} = (w_1Y_1+w_2Y_2+\ldots+w_jY_j)/j$, and $\overline{X}_{wj} = (w_1X_1+w_2X_2+\ldots+w_jX_j)/j$ for $i = 1,2,\ldots,j$ and $j = 1,2,\ldots.$

$$\hat{a}_j = \frac{-\sum_{i=1}^{j} (X_i - \overline{X}_{wj})\, Y_i\, w_i}{\sum_{i=1}^{j} (X_i - \overline{X}_{wj})^2\, w_i} \qquad (3.8)$$

$$\hat{b}_j = \frac{1}{1-\hat{a}_j} \exp(\overline{Y}_{wj} + \hat{a}_j \overline{X} w_j) \qquad (3.9)$$

for $j = 2,3,\ldots.$ As in the previous model, the instantaneous failure rate estimate given by equation 3-1 or 3-2 for j=1 is used for the first phase.

B. FAILURE DISCOUNTING

Another modification that can be applied to this model is fractional failure removal known as failure discounting. Some investigations have been made on the application of failure discounting to discrete reliability growth models [refs. 5 and 6]. What follows here is a brief description of failure discounting from previous work.

Testing conducted during the initial stages of a particular system often indicates a low reliability. Generally, weaknesses in the configuration of the system or defects in the quality of its components cause system failure. Test designs are established so that the cause for these failures can be identified and corrected. Theoretically, then, as a weakness or a defect is identified and, hopefully, corrected the probability of that particular weakness or defect reoccurring should be reduced. This reduction in the probability of occurrence of a certain failure cause leads to improved system reliability. This concept is fully utilized in failure discounting.

> In order to effectively discount previous failures it is critical that the cause of the failure be properly identified. The level of detail that one wishes to ascribe to this identification process is dependent upon the type of system being evaluated and the purpose of the test. If a complex system is being evaluated then a failure cause may be failure of a certain component or sub component. The precise element that caused system failure is not critical but the ability to assign a failure cause to each system failure is [critical]. Correctly determining failure cause is very difficult. particularly when dealing with complex systems. Therefore, it is conceivable that the design changes do not improve system reliability. In fact, these changes may even degrade reliability. To apply failure discounting procedures described below, one must be able to assign failure causes to every system failure... [ref. 5: pp. 3-4].

In this model the number of failures used in equation (3.1) is adjusted by removing a fixed fraction of a failure each time a predetermined amount of test time has past without reoccurrence of the failure cause. To employ the standard or straight percent discounting method one must specify two parameters. These are the fraction of a failure to be removed, f, and the discount interval, Treq. The adjusted number of failures due to a single cause is calculated as

$$F_{adj} = F \bullet (1-f)^{INT(Tsf/Treq)} \tag{3.10},$$

where Tsf = Time Since Last Failure for the cause and where INT(x) is a function that returns the integer-part of x by truncating any fractional part. When a failure due to the same cause reoccurs, Tsf is reset to 0, and any previously removed fractions are restored.

A new number of failures for each phase is calculated as the sum of the adjusted failures for each cause in that phase. These adjusted numbers of failures are used in equation (3.1) and the results used in equations (3.3) and (3.4) as described above.

An example of the application of the standard method of failure discounting will clarify this process. Consider the data in Table I. This fictitious data is intended solely to illustrate this discounting

Table I: STANDARD METHOD OF FAILURE DISCOUNTING

PHASE	TIME	ACTUAL FAILURES	CAUSE	ADJUSTED FAILURES
1	2	1	X	1.00
2	3	2	Y	0.50 + 1.00 = 1.50
3	7	3	X	2.00 + 0.25 = 2.25

f = 0.5, Treq = 3

method. Assume that improvements are made after each failure. Thus Table I represents three phases of a test-fix-test scenario with two possible causes of failure, X and Y. In this example Treq = 3 mission units and f = .5

The second failure, attributed to cause Y, terminates phase two. At that time the failure due to cause X has had 3 mission units without a reoccurrence of that cause so the failure discounting formula is applied:

$F_{adj} = F(1-f)^{INT(Tsf/Treq)} = 1(1-0.5)^{INT(3/3)} = 0.50$

Thus at the end of phase two the failure that ended phase one is counted as one-half of a failure. Phase three ends with a reoccurrence of failure cause X. The value for Tsf is reset to 0 resulting in the restoring of all failures due to cause X to full value. However there have been 7 mission units since a failure due to cause Y. Therefore the discounting formula is applied:

$$F_{adj} = F(1-f)^{INT(Tsf/Treq)} = 1(1-0.5)^{INT(7/3)} = (0.50)^2 = 0.25$$

IV. SIMULATION METHODOLOGY

Monte Carlo simulation methods are used to analyze the models discussed in this thesis. The simulation is written for a micro-computer in the programming language Fortran (see appendix B for listing). A Monte-Carlo Simulation was used for several reasons. First, the data can be generated from a known distribution; in this case exponential failure times. Second, the parameters, i.e. failure rate, can be controlled. Third, the results are reproducible if the same seed is used. This allows the parameters for the failure discounting method and regression weights to be altered and compared using the same data. Finally the data is very easy and inexpensive to generate, allowing many repetitions of the same experiment. This allows statistics such as the average and mean square error (MSE) to be calculated very easily.

A. PROGRAM

In the simulation, the exponential failure times were calculated stochastically using a transform on the output of a uniform pseudo-random number generator. Times for each cause were generated and then the minimum time was selected as the next failure and its cause was recorded as the cause of the failure. This is similar to the "fixed phase reliability option" used by Chandler [ref. 5]. Rather than generating a failure time for each item on test, the time until the first of n items fails was generated. This time is the minimum of n exponential random variables with failure rate λ. The program keeps track of total test time in each phase, the total time since the last failure for each cause, and the adjusted as well as actual number of failures for each cause in each phase. These values are utilized to estimate the failure rate for each phase using the four models discussed in the previous chapters, the cumulative AMSAA, Modified AMSAA, Modified AMSAA with regression weights, and Modified AMSAA with failure discounting. The program calculates the average and MSE for the estimate of each model over a number of replications. Five hundred replications were used for all cases simulated in this thesis. Appendix II gives a more detailed description of the program and its use.

B. FAILURE RATE PATTERNS

Eight patterns of reliability growth and non-growth are simulated to evaluate the four models over a range of possible cases. The Eight failure rate patterns simulated are taken from the eight reliability patterns used by Chandler [ref. 5: pp. 19-30]. These reliabilities are converted into failure rates assuming a one hour nominal mission time. This assumption leads to the relationship $R = e^{-\lambda}$ or $\lambda = -\ln R$. In this way, Chandler's matrices of Reliability were converted to matrices of Failure rates in failures per hour. These eight matrices were used as inputs to the simulation and are shown in Tables II through IX.

The failure rate patterns are depicted in Figures 1 through 8. In all of the following descriptions, reliability growth is analogous to a decrease in the failure rate. Figure 1 depicts a pattern of non-concave reliability growth which may not be unusual in situations where the exact method or technology required to correct a failure causing defect is not immediately available, but, as the systems evolves and the personnel become more familiar with it, the failure correction process proceeds more efficiently. This pattern is convex in the reliability function, $e^{-\lambda t}$.

Figure 2 represents a pattern of increasing, then decreasing, then finally increasing reliability. This type of pattern can result from experimental systems where the results of design changes may introduce new modes of failure when they are implemented.

Figure 3 represents a pattern of growth which stagnates for a few phases before finally achieving mature reliability.

Figures 4 and 5 depict conventional reliability growth patterns one would expect to encounter when evaluating the majority of systems. Pattern 4 ultimately attains a higher reliability (lower failure rate) than pattern 5.

Figures 6,7, and 8 represent constant system reliabilities that are moderately high medium and low respectively.

These eight patterns were simulated over 500 replications for each of the four models, each of the four discounting parameters and each of the four weighting schemes.

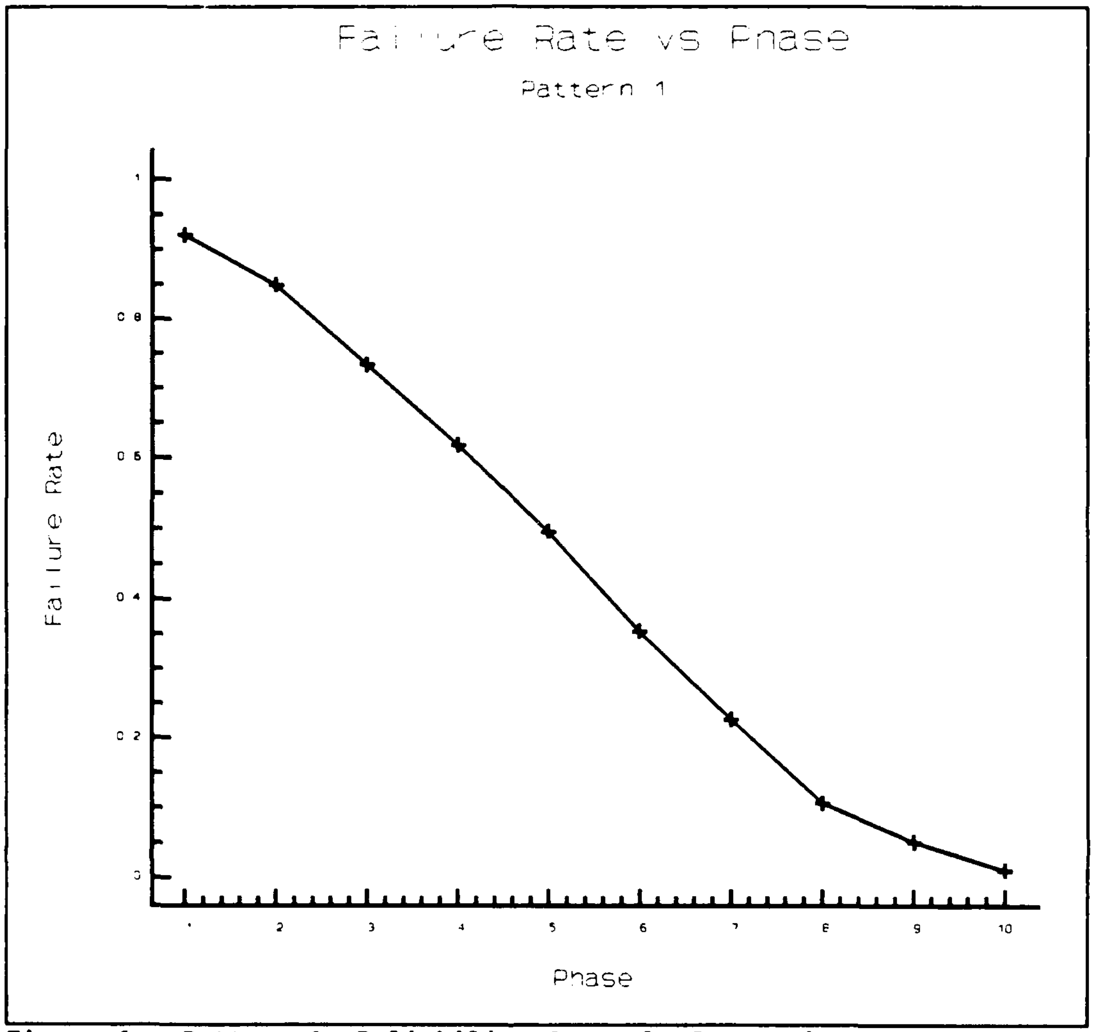


Figure 1: Pattern 1, Reliability Convexly Increasing

Table II: PATTERN 1 USER INPUTS

	Phase Failure Rate									
Cause	1	2	3	4	5	6	7	8	9	10
1	.1625	.1508	.1054	.0943	.0726	.0513	.0305	.0101	.0101	.0020
2	.1744	.1625	.1393	.1054	.0834	.0513	.0305	.0101	.0101	.0020
3	.1863	.1744	.1508	.1278	.1054	.0726	.0408	.0202	.0101	.0020
4	.1863	.1744	.1625	.1393	.1165	.0834	.0619	.0253	.0101	.0020
5	.2107	.1863	.1744	.1508	.1165	.0943	.0619	.0398	.0101	.0020

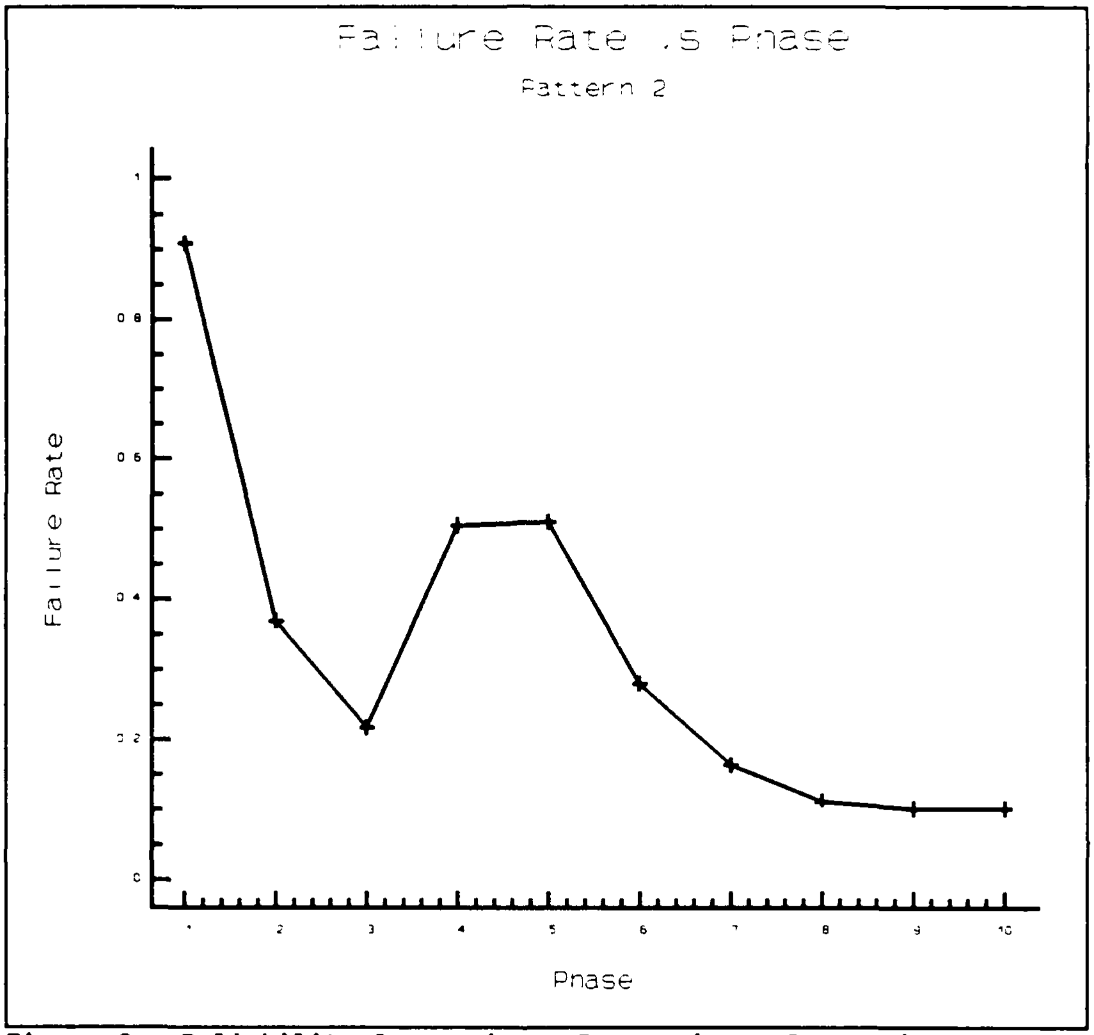


Figure 2: Reliability Increasing - Decreasing - Increasing

Table III: PATTERN 2 USER INPUTS

	Phase Failure Rate									
Cause	1	2	3	4	5	6	7	8	9	10
1	.0202	.0202	.0101	.0101	.0101	.0101	.0101	.0101	.0101	.0101
2	.0513	.0305	.0202	.0202	.2485	.0726	.0202	.0101	.0101	.0101
3	.1985	.0726	.0408	.3285	.1054	.0513	.0305	.0202	.0202	.0202
4	.2231	.0834	.0408	.0408	.0408	.0408	.0408	.0305	.0202	.0202
5	.4155	.1625	.1054	.1054	.1054	.1054	.0619	.0408	.0408	.0408

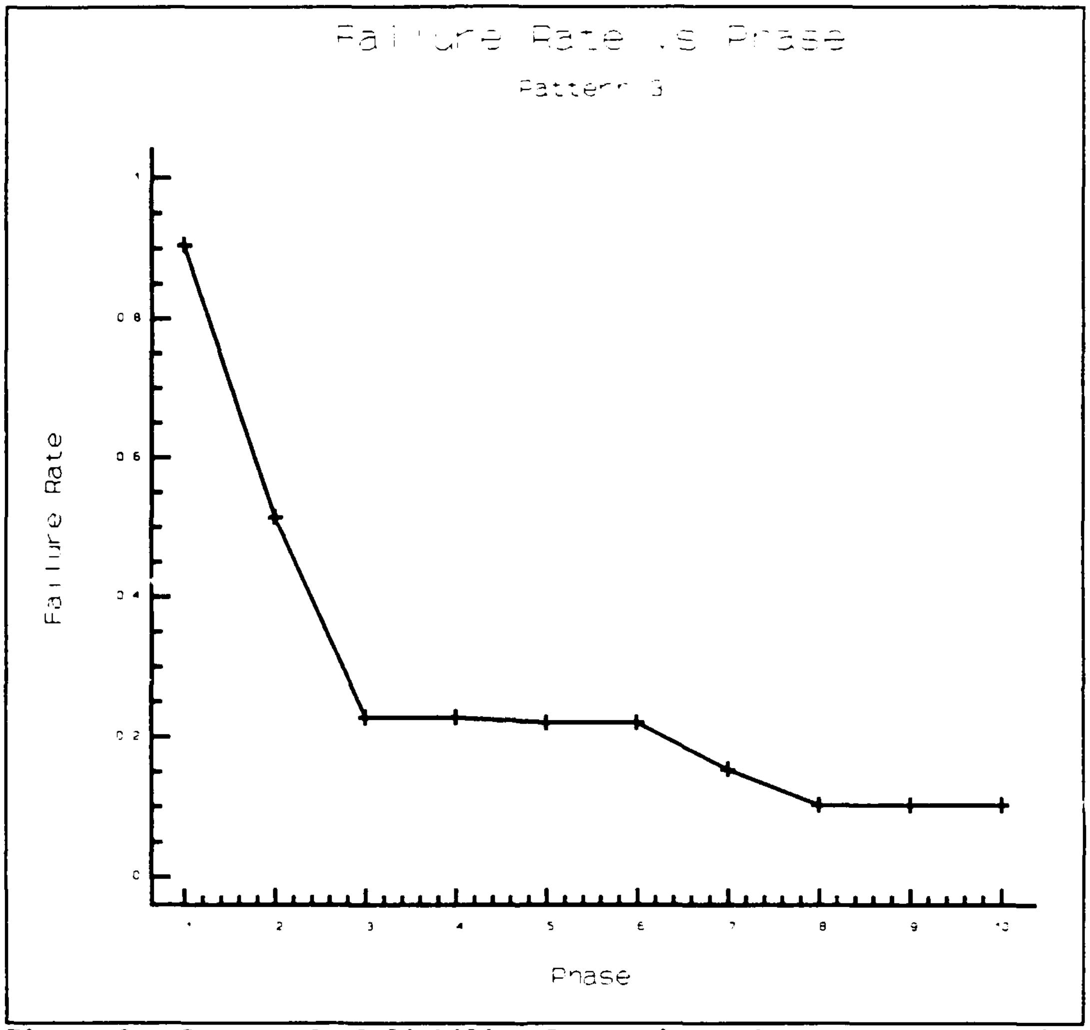


Figure 3: Pattern 3, Reliability Increasing - Constant - Increasing

Table IV: PATTERN 3 USER INPUTS

Cause	Phase Failure Rate 1	2	3	4	5	6	7	8	9	10
1	.1508	.0726	.0305	.0305	.0202	.0202	.0202	.0101	.0101	.0101
2	.1508	.0726	.0305	.0305	.0202	.0202	.0202	.0101	.0101	.0101
3	.1508	.0726	.0305	.0305	.0202	.0202	.0202	.0101	.0101	.0101
4	.1508	.0726	.0305	.0305	.0202	.0202	.0202	.0101	.0101	.0101
5	.3011	.2231	.1054	.1054	.1393	.1393	.0726	.0619	.0619	.0619

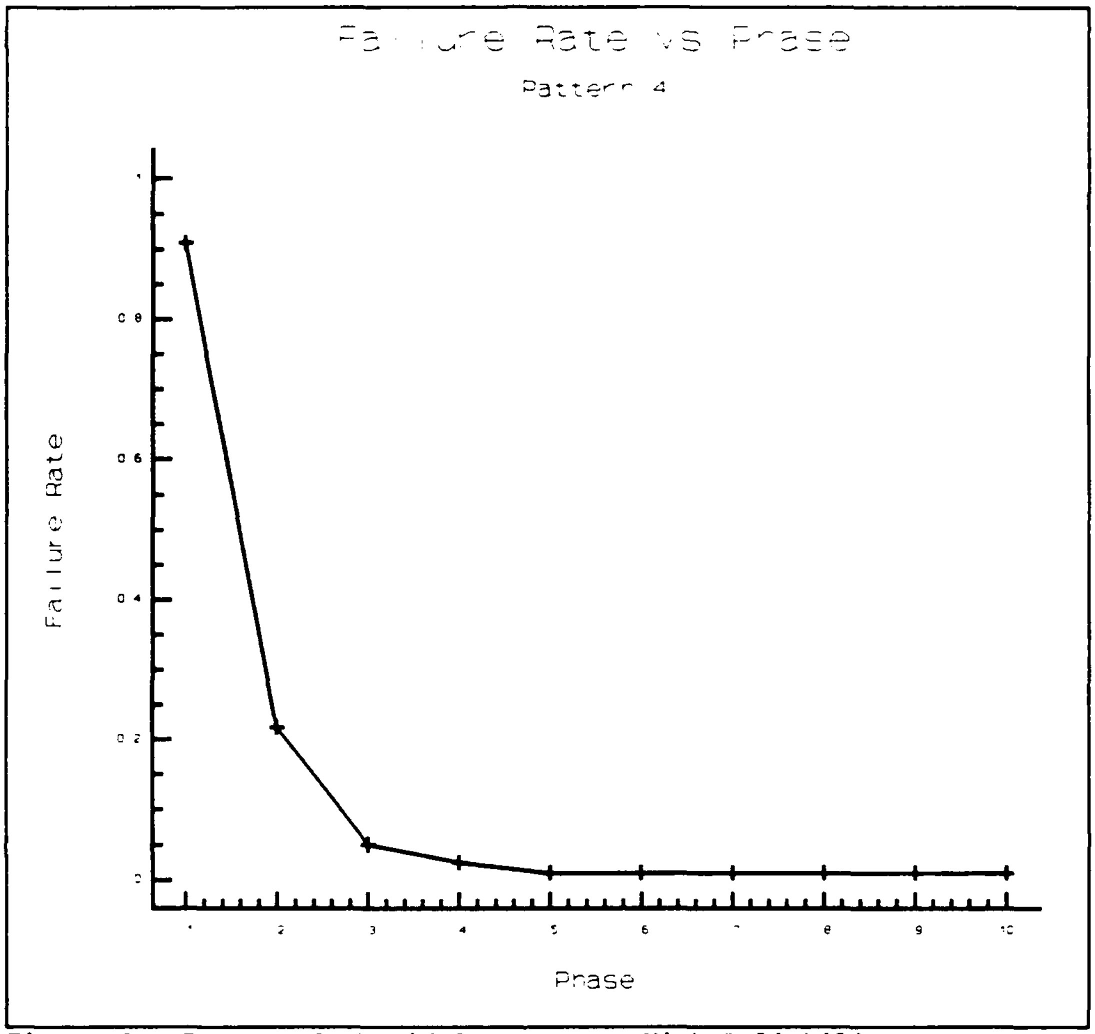


Figure 4: Pattern 4, Rapid Increase to High Reliability

Table V: PATTERN 4 USER INPUTS

	Phase Failure Rate									
Cause	1	2	3	4	5	6	7	8	9	10
1	.0202	.0101	.0101	.0050	.0020	.0020	.0020	.0020	.0020	.0020
2	.0513	.0202	.0101	.0050	.0020	.0020	.0020	.0020	.0020	.0020
3	.1985	.0408	.0101	.0050	.0020	.0020	.0020	.0020	.0020	.0020
4	.2231	.0408	.0101	.0050	.0020	.0020	.0020	.0020	.0020	.0020
5	.4155	.1054	.0101	.0050	.0020	.0020	.0020	.0020	.0020	.0020

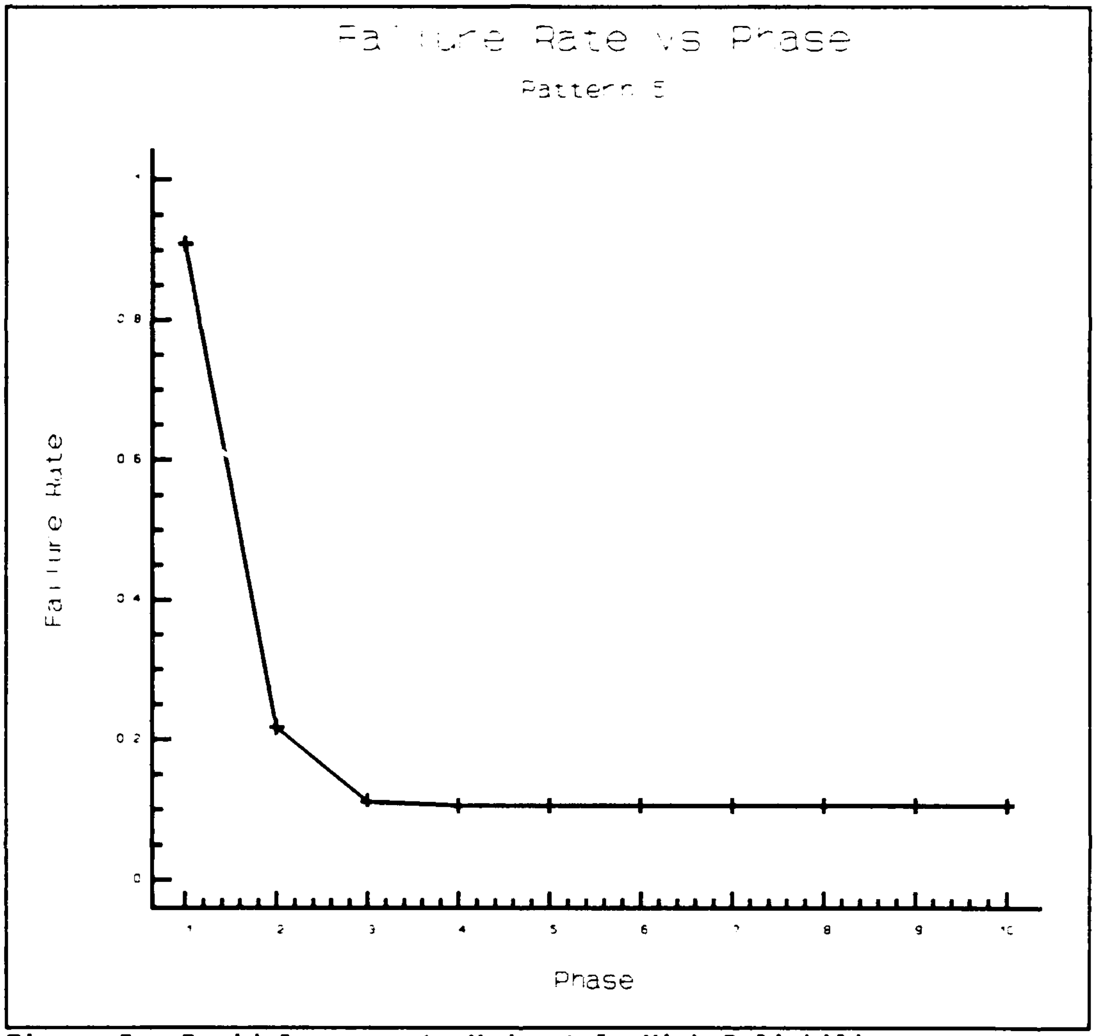


Figure 5: Rapid Increase to Moderately High Reliability

Table VI: PATTERN 5 USER INPUTS

Cause	Phase Failure Rate 1	2	3	4	5	6	7	8	9	10
1	.0202	.0101	.0101	.0101	.0101	.0101	.0101	.0101	.0101	.0101
2	.0513	.0202	.0101	.0101	.0101	.0101	.0101	.0101	.0101	.0101
3	.1985	.0408	.0202	.0202	.0202	.0202	.0202	.0202	.0202	.0202
4	.2231	.0408	.0305	.0253	.0253	.0253	.0253	.0253	.0253	.0253
5	.4155	.1054	.0408	.0398	.0398	.0398	.0398	.0398	.0398	.0398

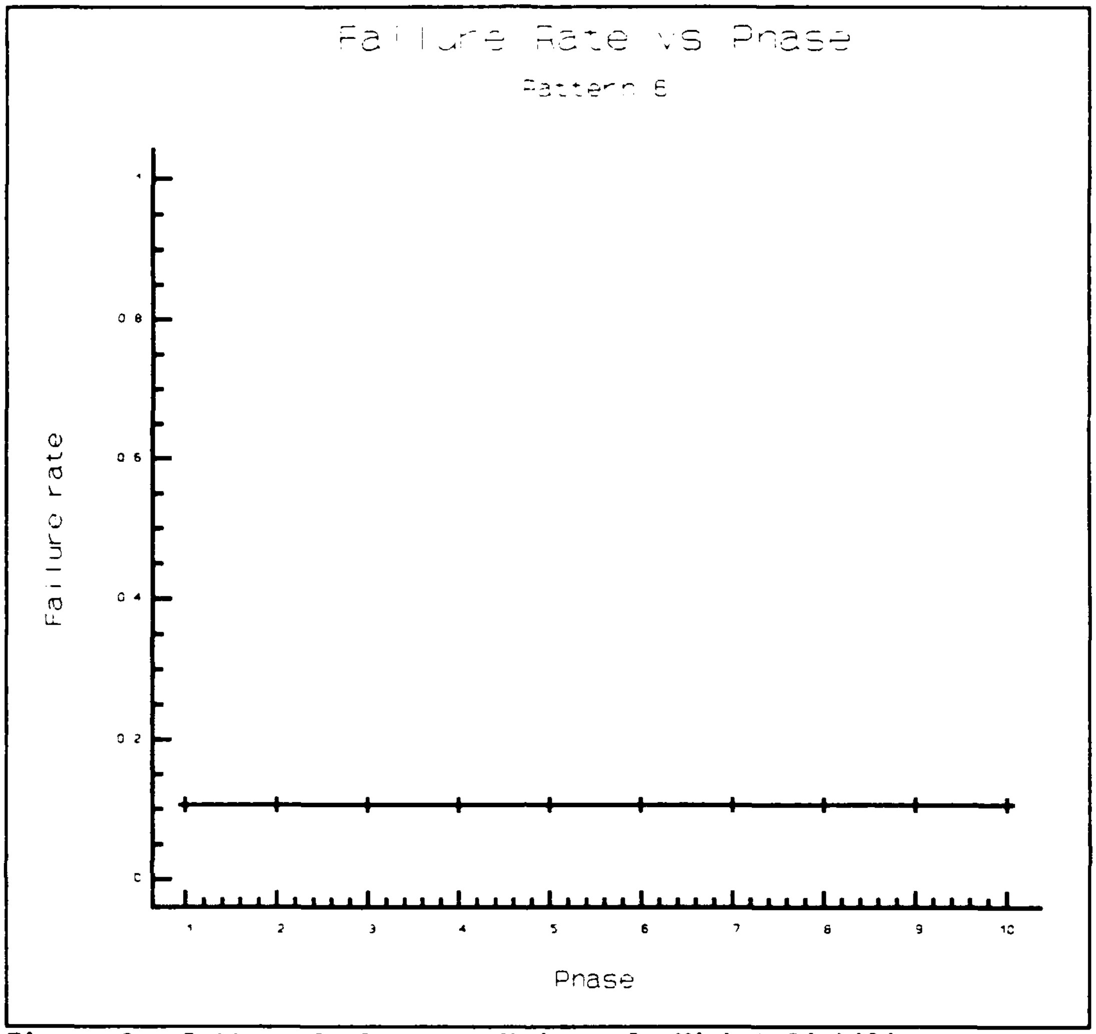


Figure 6: Pattern 6, Constant Moderately High Reliability

Table VII: PATTERN 6 USER INPUTS

	Phase Failure Rate									
Cause	1	2	3	4	5	6	7	8	9	10
1	.0101	.0202	.0202	.0253	.0305	.0305	.0253	.0202	.0202	.0101
2	.0202	.0202	.0253	.0305	.0101	.0101	.0305	.0253	.0202	.0202
3	.0202	.0253	.0305	.0101	.0202	.0202	.0101	.0305	.0253	.0202
4	.0253	.0305	.0101	.0202	.0202	.0202	.0202	.0101	.0305	.0253
5	.0305	.0101	.0202	.0202	.0253	.0253	.0202	.0202	.0101	.0305

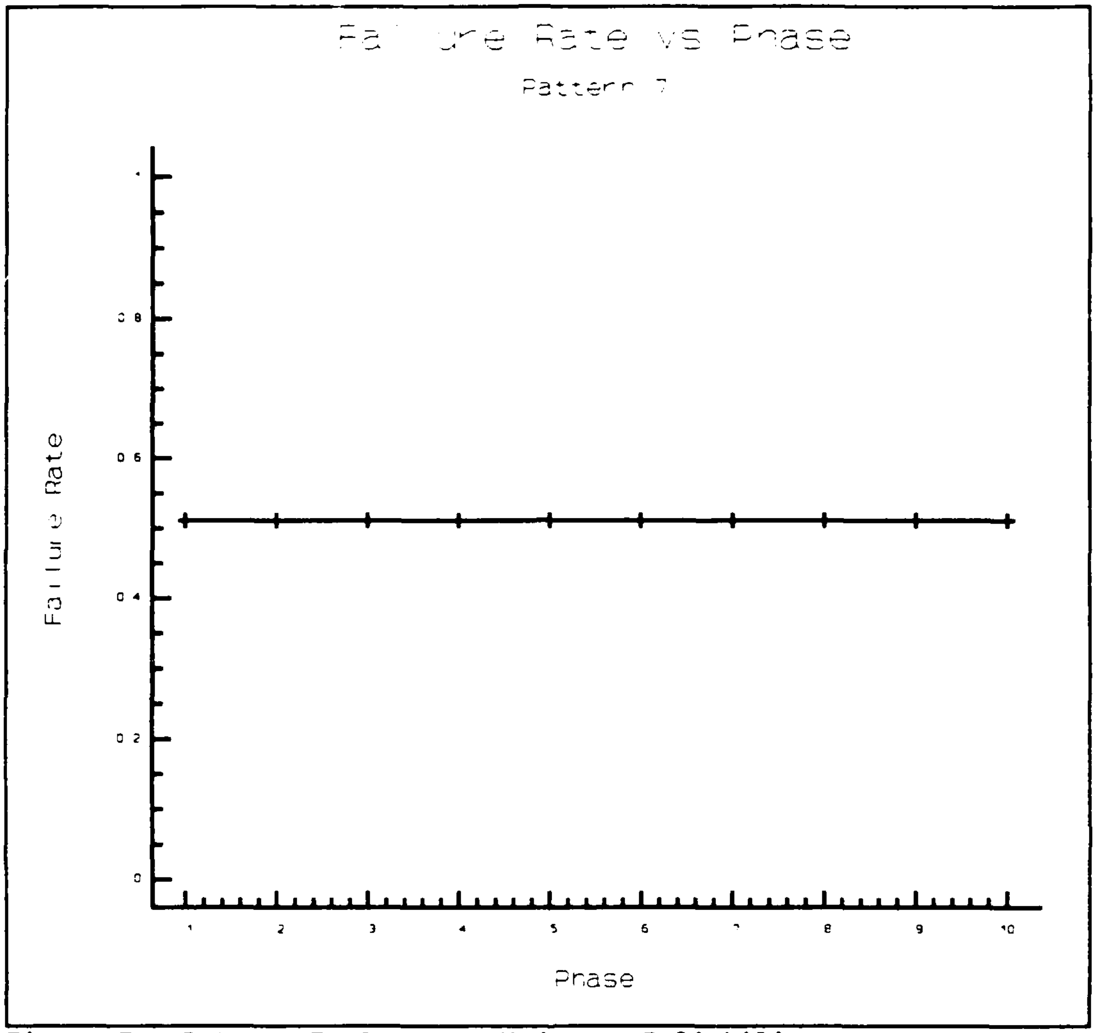


Figure 7: Pattern 7, Constant Moderate Reliability

Table VIII: PATTERN 7 USER INPUTS

Cause	Phase Failure Rate 1	2	3	4	5	6	7	8	9	10
1	.0101	.2485	.1054	.0408	.1054	.1054	.0408	.1054	.2485	.0101
2	.2485	.1054	.0408	.1054	.0101	.0101	.1054	.0408	.1054	.2485
3	.1054	.0408	.1054	.0101	.2485	.2485	.0101	.1054	.0408	.1054
4	.0408	.1054	.0101	.2485	.1054	.1054	.2485	.0101	.1054	.0408
5	.1054	.0101	.2485	.1054	.0408	.0408	.1054	.2485	.0101	.1054

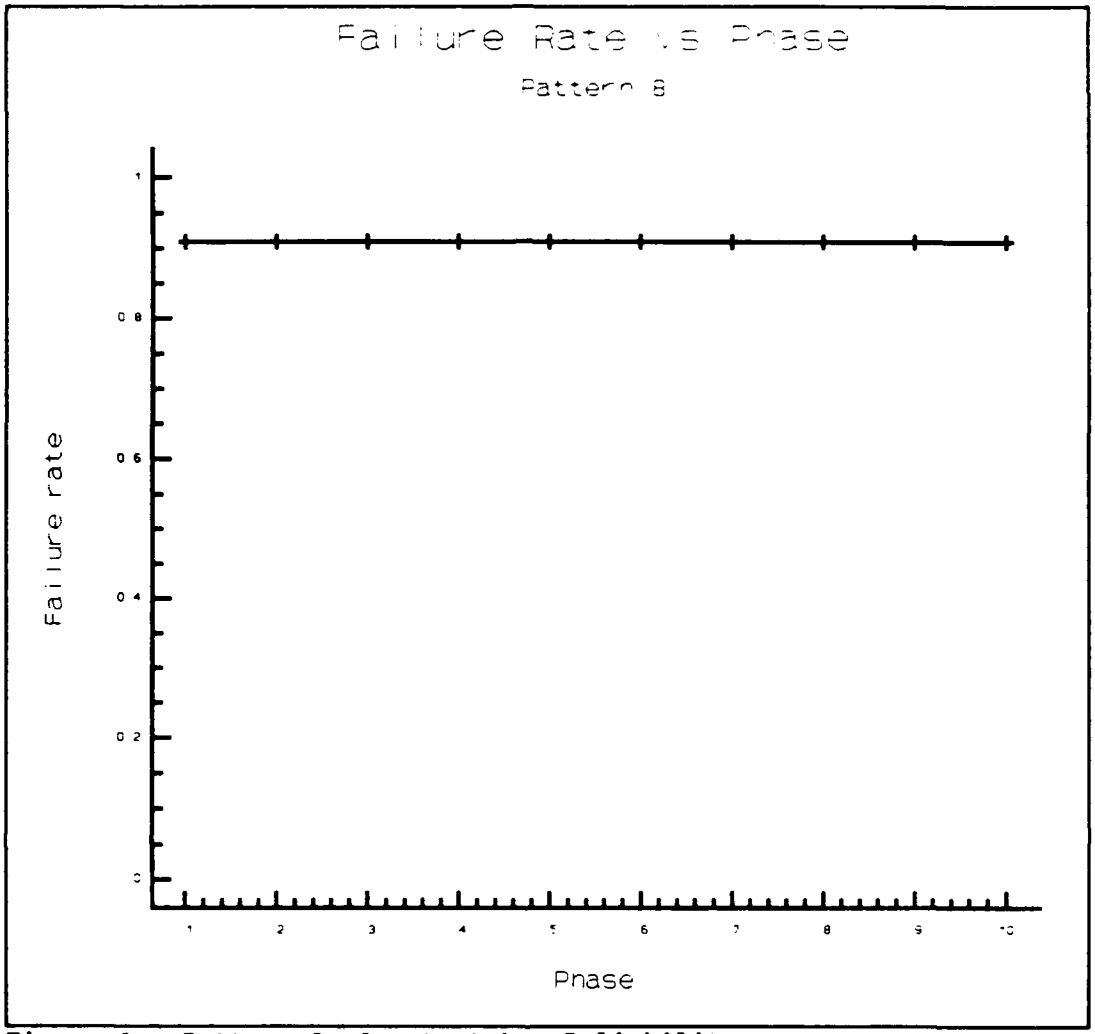


Figure 8: Pattern 8, Constant Low Reliability

Table IX: PATTERN 8 USER INPUTS

	Phase Failure Rate									
Cause	1	2	3	4	5	6	7	8	9	10
1	.0202	.0513	.1985	.2231	.4155	.4155	.2231	.1985	.0513	.0202
2	.0513	.1985	.2231	.4155	.0202	.0202	.4155	.2231	.1985	.0513
3	.1985	.2231	.4155	.0202	.0513	.0513	.0202	.4155	.2231	.1985
4	.2231	.4155	.0202	.0513	.1985	.1985	.0513	.0202	.4155	.2231
5	.4155	.0202	.0513	.1985	.2231	.2231	.1985	.0513	.0202	.4155

V. INTERPRETATION OF RESULTS

This chapter describes the results of the simulation runs for the eight failure patterns. Four runs, designated E, F, G and H, were conducted for each pattern and each model. The parameters used in failure discounting and the weighting scheme used were varied over these four runs. The performance of the four models for these runs are reduced to two graphs, for each pattern and each run, to allow comparison. The first graph plots the average of the estimates of λ_{TTk} over the 500 replications and the true failure rate against the phase. The second graph plots the Mean square error (MSE) for the estimates against the phase. The MSE is used to compare the performance of the models relative to each other. The MSE is calculated for each model and each phase by

$$MSE = \Sigma(\hat{\lambda}_{TTk}-\lambda_k)^2/500 \qquad (5.1)$$

where the summation is over the 500 replications. Due to space constraints, the graphical results of only a few of the more representative runs are included in this chapter. The graphs for all runs conducted are included in Appendix C.

This interpretation is organized according to the reliability growth patterns introduced in the previous chapter. These patterns will be referred to by their numerical designator. The numerals are listed on the figures in the preceding chapter and are summarized below:

- Pattern 1 - Convexly increasing reliability
- Pattern 2 - Reliability increasing rapidly, then decreasing then increasing again

- Pattern 3 - Reliability increasing rapidly, then constant, then increasing again

- Pattern 4 - Rapidly increasing to high reliability

- Pattern 5 - Rapidly increasing to moderately high reliability

- Pattern 6 - Constant moderately high reliability

- Pattern 7 - Constant moderate reliability

- Pattern 8 - Constant low reliability

Recall that in all of the following graphs the standard AMSAA model was not discounted nor weighted. Each of these Patterns was evaluated for four sets of regression weights and four sets of discounting parameters as summarized in Tables X and XI respectively.

Table X: REGRESSION WEIGHTS USED

Run	1	2	3	4	5	6	7	8	9	10
E	.050	.224	.368	.473	.549	.607	.652	.688	.717	.741
F	.250	.500	.620	.707	.758	.794	.820	.841	.857	.871
G	.500	.707	.794	.841	.871	.891	.905	.917	.925	.933
H	.100	.100	.110	.120	.130	.140	.150	.700	.800	.900

Notes:
1. E, F, and G used weighting method 1, $w_i^* = f^{1/i}$, $f = w_1^*$.
2. H used weighting method 2, user selected weights.
3. All w_i^* above are divided by the sum of weights when used in the regression equations to ensure that weights always sum to one.

Table XI: DISCOUNTING PARAMETERS USED

Run	Fraction	Time Required Between Discounting
E	.50	3
F	.50	15
G	.25	3
H	.25	15

A. RAPIDLY INCREASING RELIABILITIES: PATTERNS 4 AND 5

Figure 9 shows the average estimates of the four models applied to

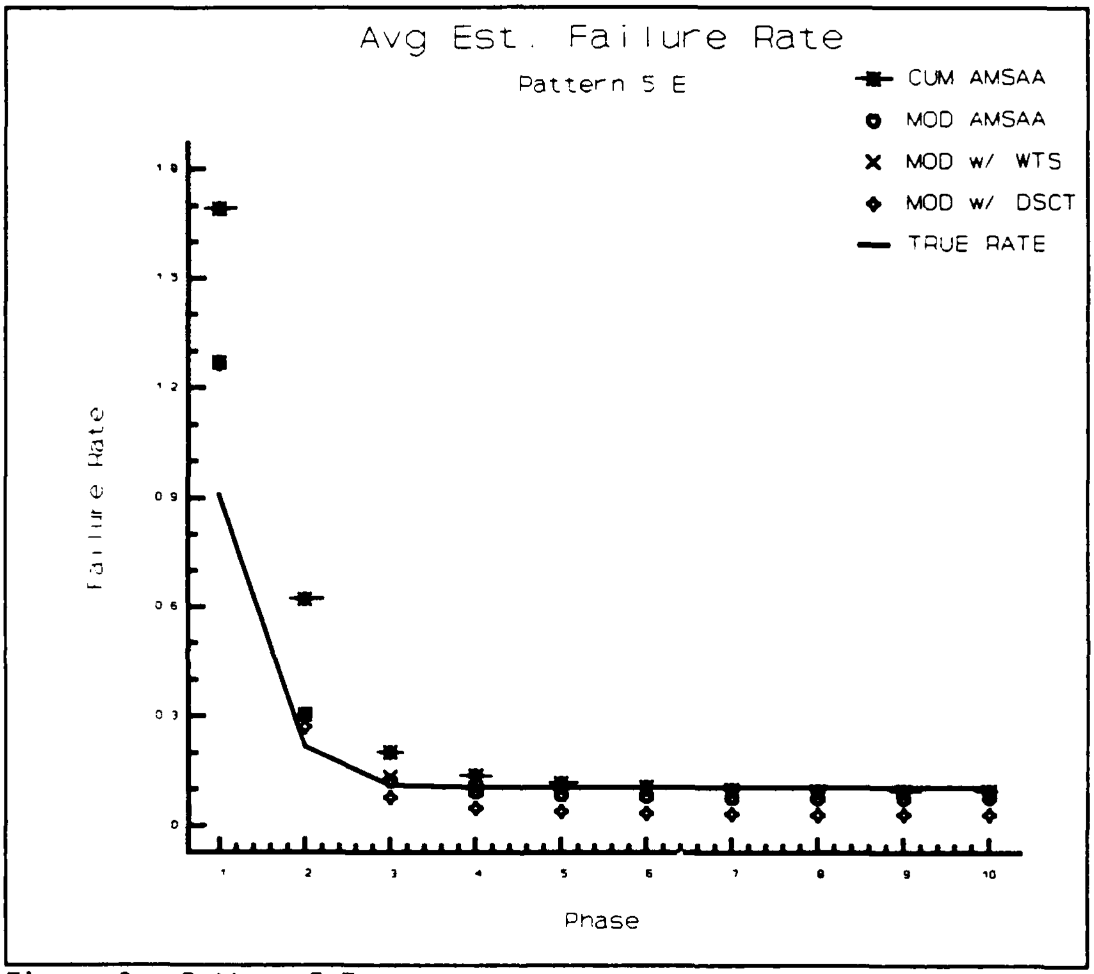


Figure 9: Pattern 5 E

pattern 5. Patterns of this type are considered typical reliability growth patterns. All four models follow this pattern well. The Modified AMSAA model is closest to the true failure rate in the earlier phases, and the Cumulative AMSAA model gives a slightly closer estimate in the last three phases.

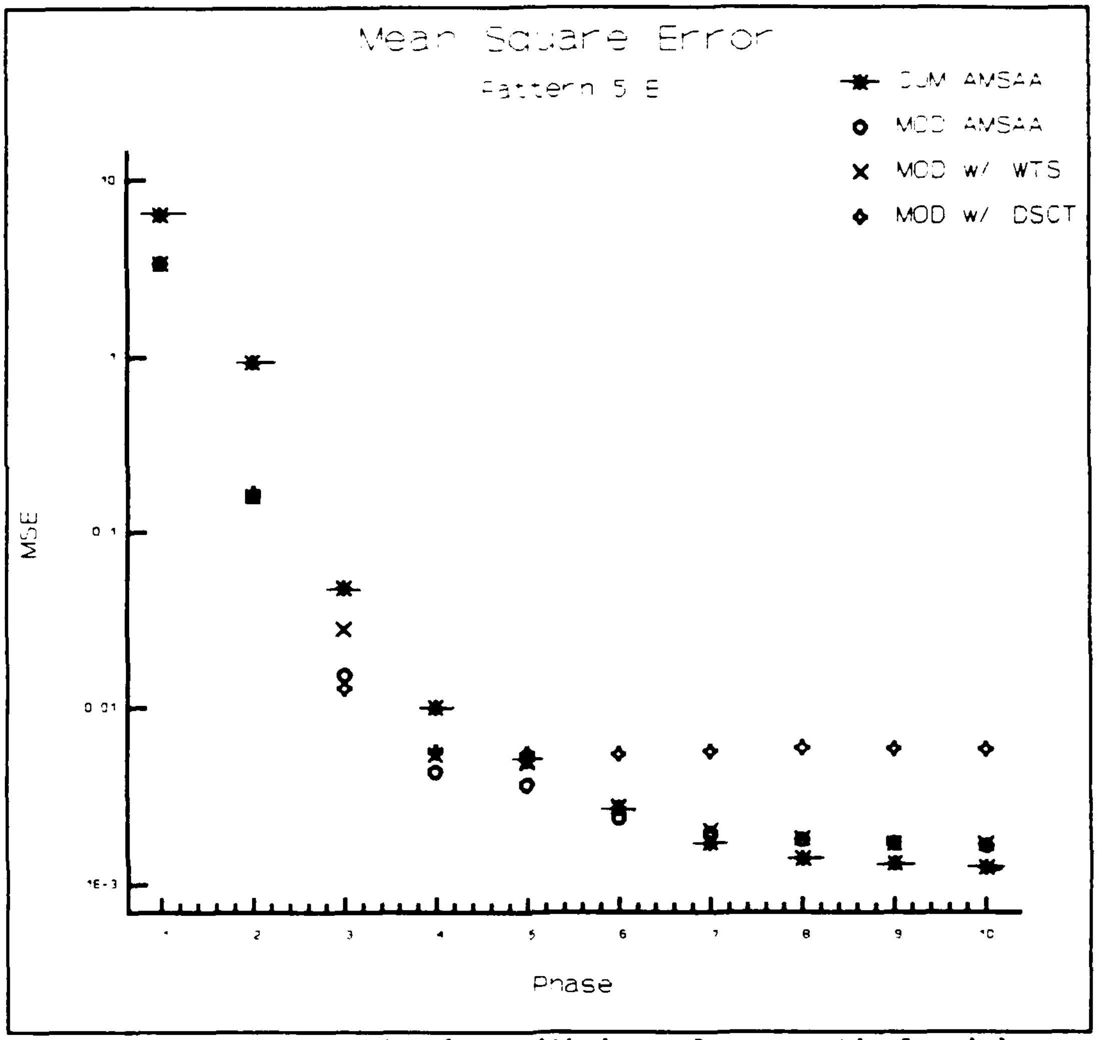


Figure 10: Pattern 5 E MSE (Logarithmic scale on vertical axis)

Figure 10 depicts the MSE of the estimates in Figure 9 (notice the logarithmic vertical scale.) The Modified AMSAA model has the lowest MSE in the early phases, and the Cumulative AMSAA has lower MSE in the last three phases. It is interesting to note how the Modified AMSAA model with failure discounting has the highest MSE in the later phases. Due to the aggressive discounting done on this run, the estimates become too optimistic and the failure rate is underestimated.

B. CONVEXLY INCREASING RELIABILITY: PATTERN 1

Figure 11 indicates the performance of the models on a nonstandard pattern of reliability growth. The Cumulative AMSAA model does not follow this pattern. The Modified AMSAA models perform the best on this type of pattern.

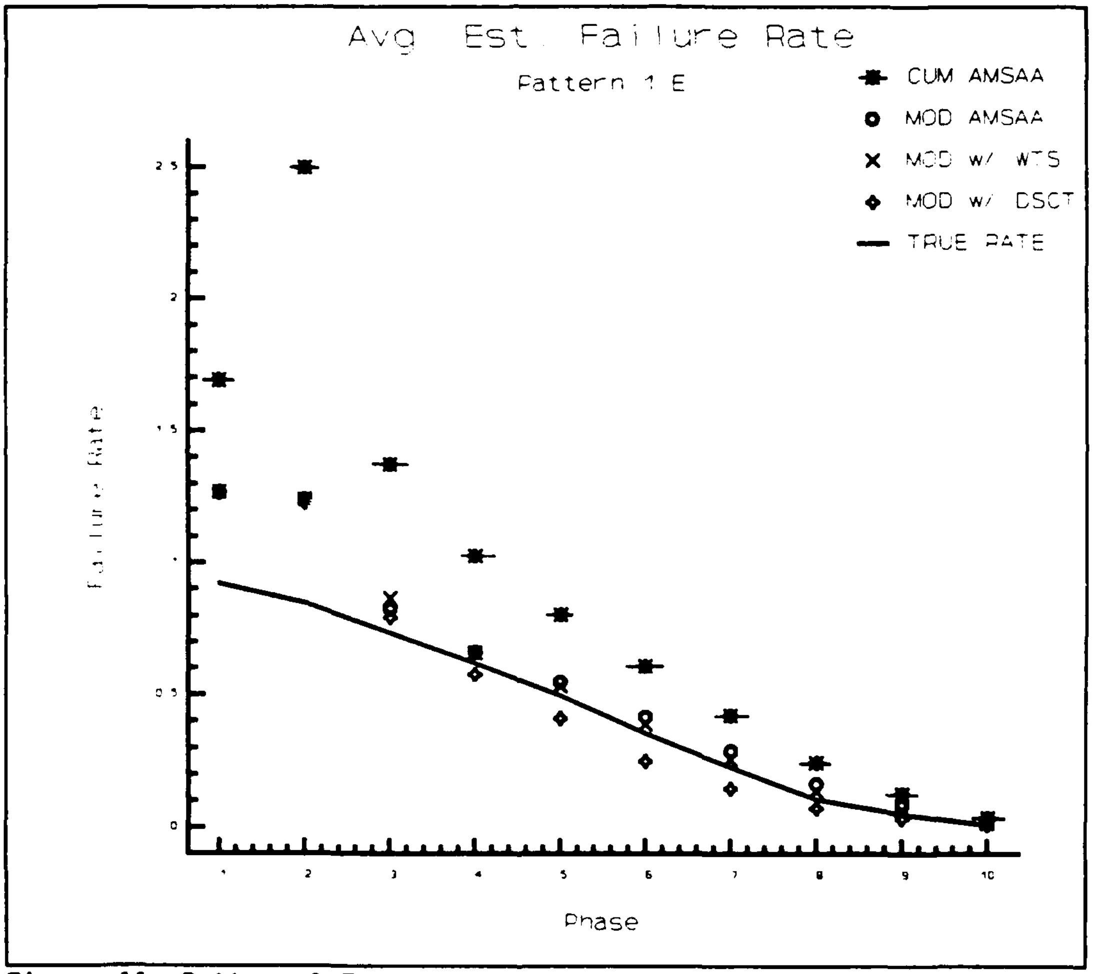


Figure 11: Pattern 1 E

As Figure 12 indicates, the Modified AMSAA with failure discounting has the lowest MSE in the last four phases. However this model appears

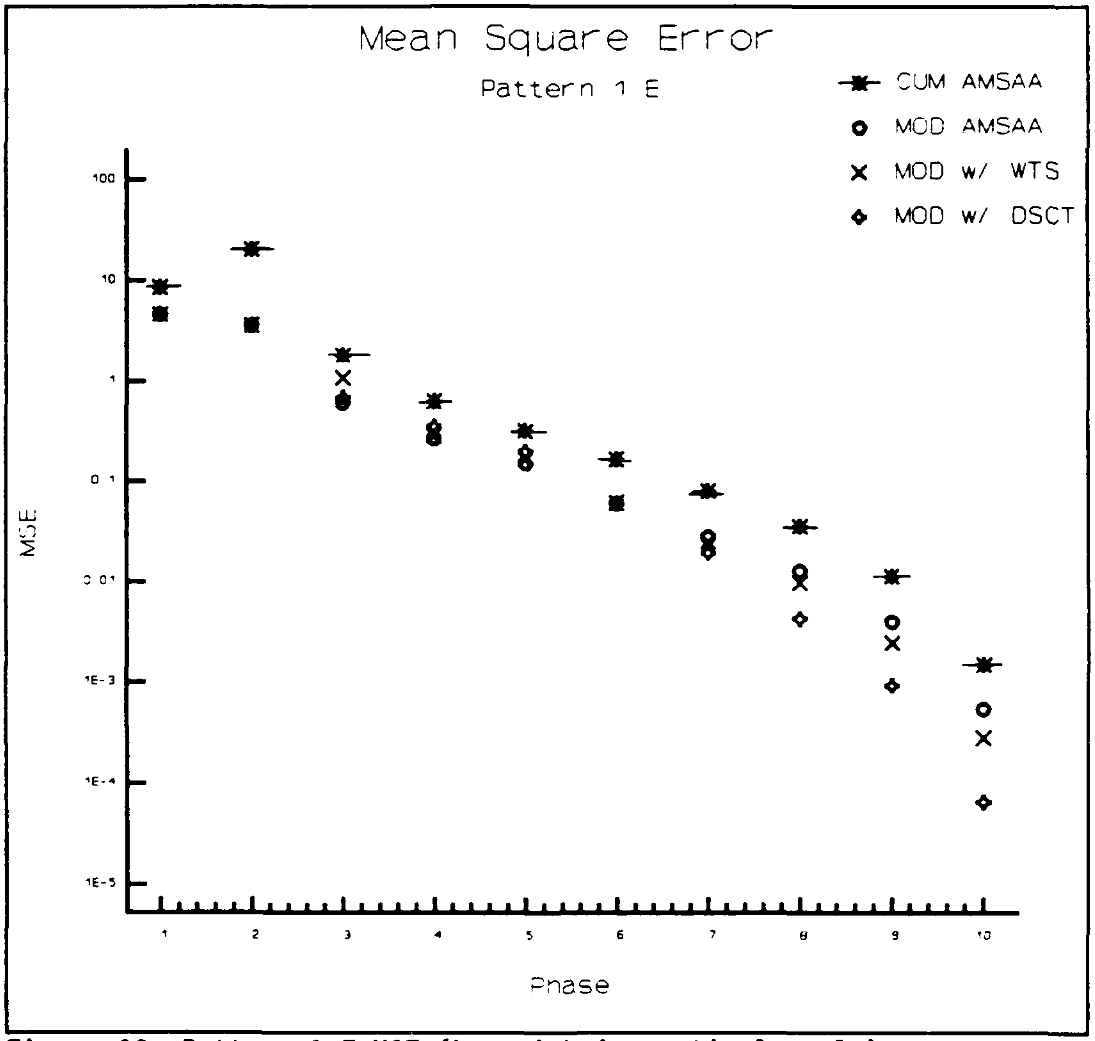


Figure 12: Pattern 1 E MSE (Logarithmic vertical scale)

to be biased toward the optimistic side. Figure 11 shows its tendency to underestimate the failure rate.

C. COMBINED RELIABILITY GROWTH AND NONGROWTH: PATTERNS 2 AND 3

Figure 13 shows the average performance of these models on Pattern 2. The Modified AMSAA model with regression weights appears to be most responsive to these direction changes. However, Figure 14 indicates that

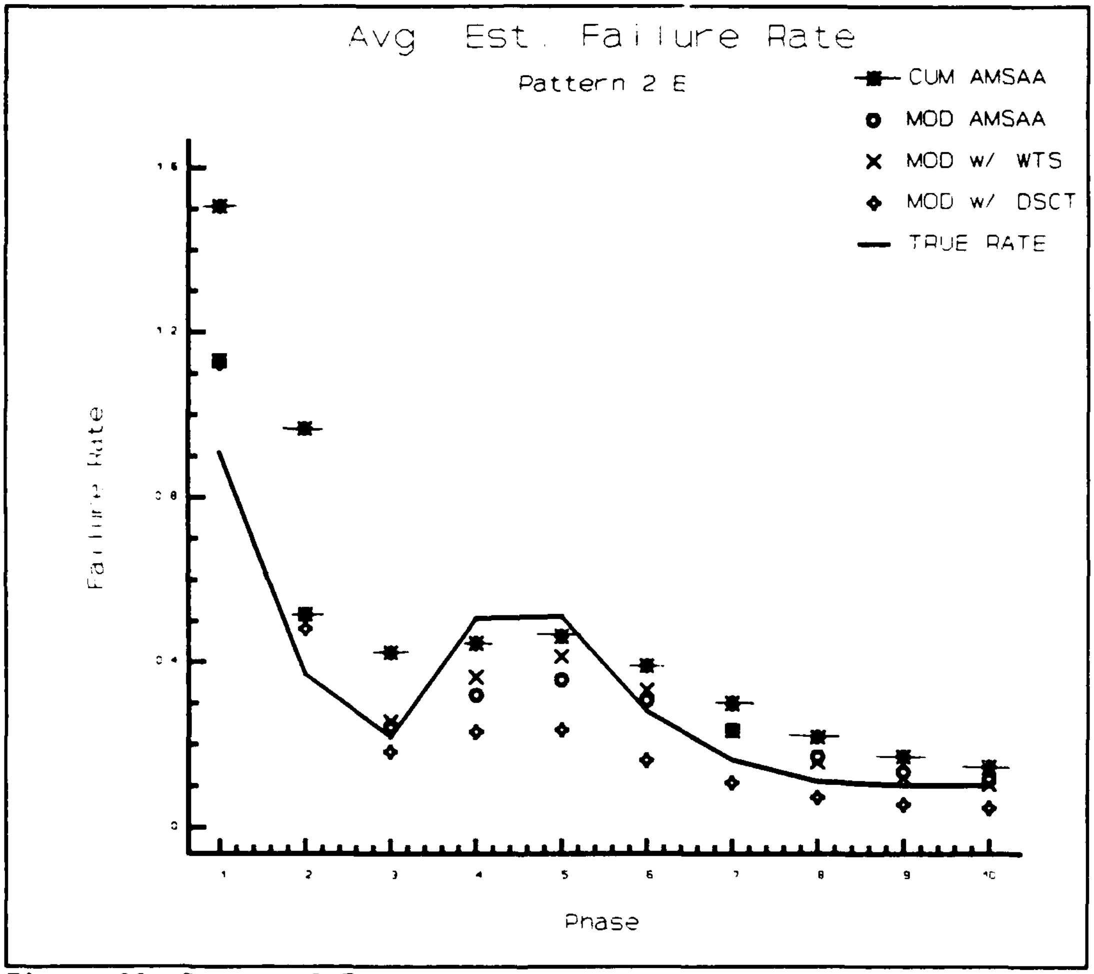


Figure 13: Pattern 2 E

the MSE of this weighted model is not smallest. This is due to the increased variability of this model. Figure 14 also shows that the discounted model has the lowest MSE in the last four phases, although it consistently underestimates the failure rate.

The unembellished Modified AMSAA model performs well in both average and MSE at all phases of this pattern. Though the Modified AMSAA model has a slightly higher MSE in the last stages, it is better suited to

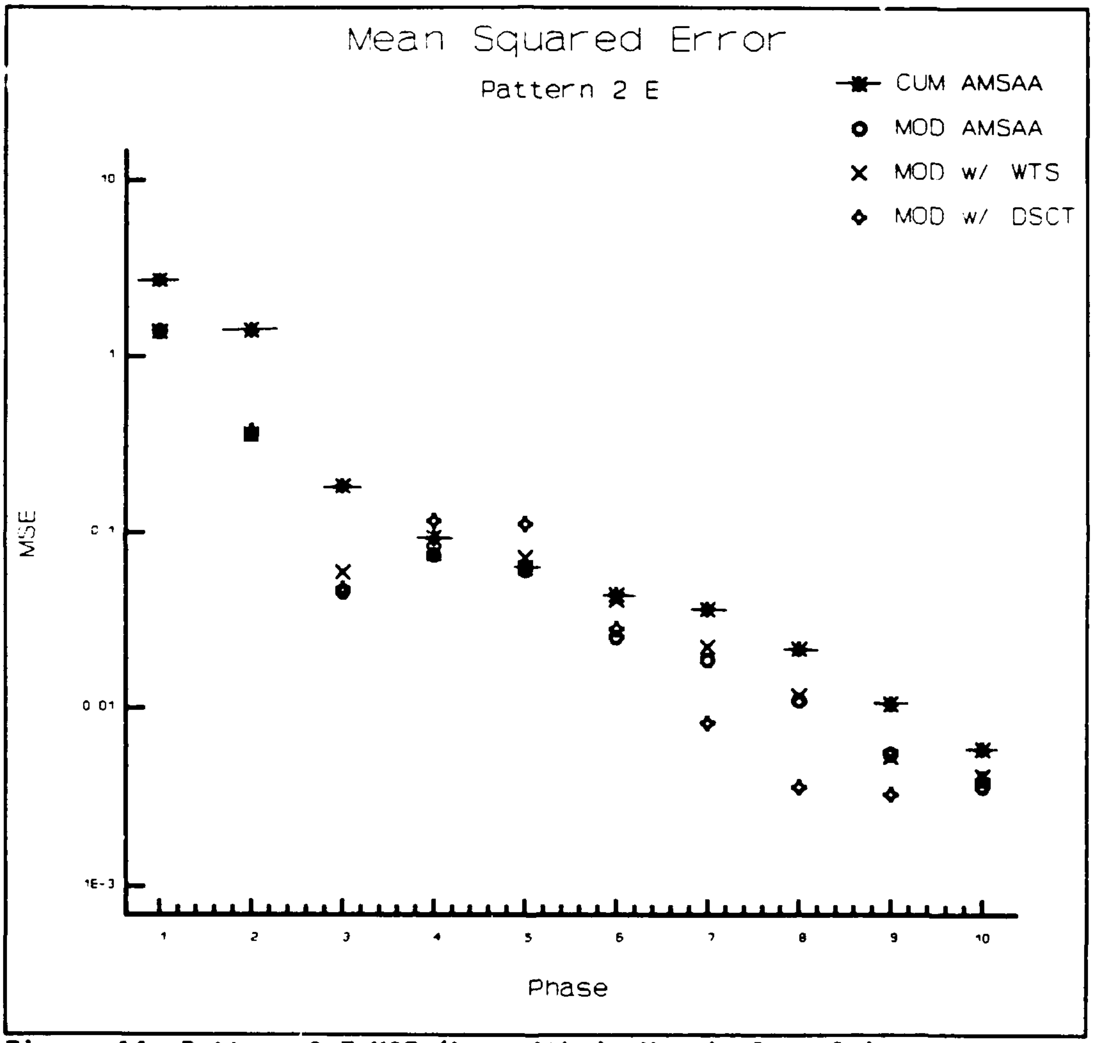


Figure 14: Pattern 2 E MSE (Logarithmic Vertical scale)

models of this sort because it is not as optimistic as the discounted version.

D. PATTERNS OF CONSTANT RELIABILITY: PATTERNS 6, 7 AND 8

Figure 15 depicts the average of the estimates when applied to Pattern 7. After the first two phases, the Modified AMSAA model follows the constant reliability pattern well. Figure 16 also shows that this model has the lowest MSE for constant failure rates.

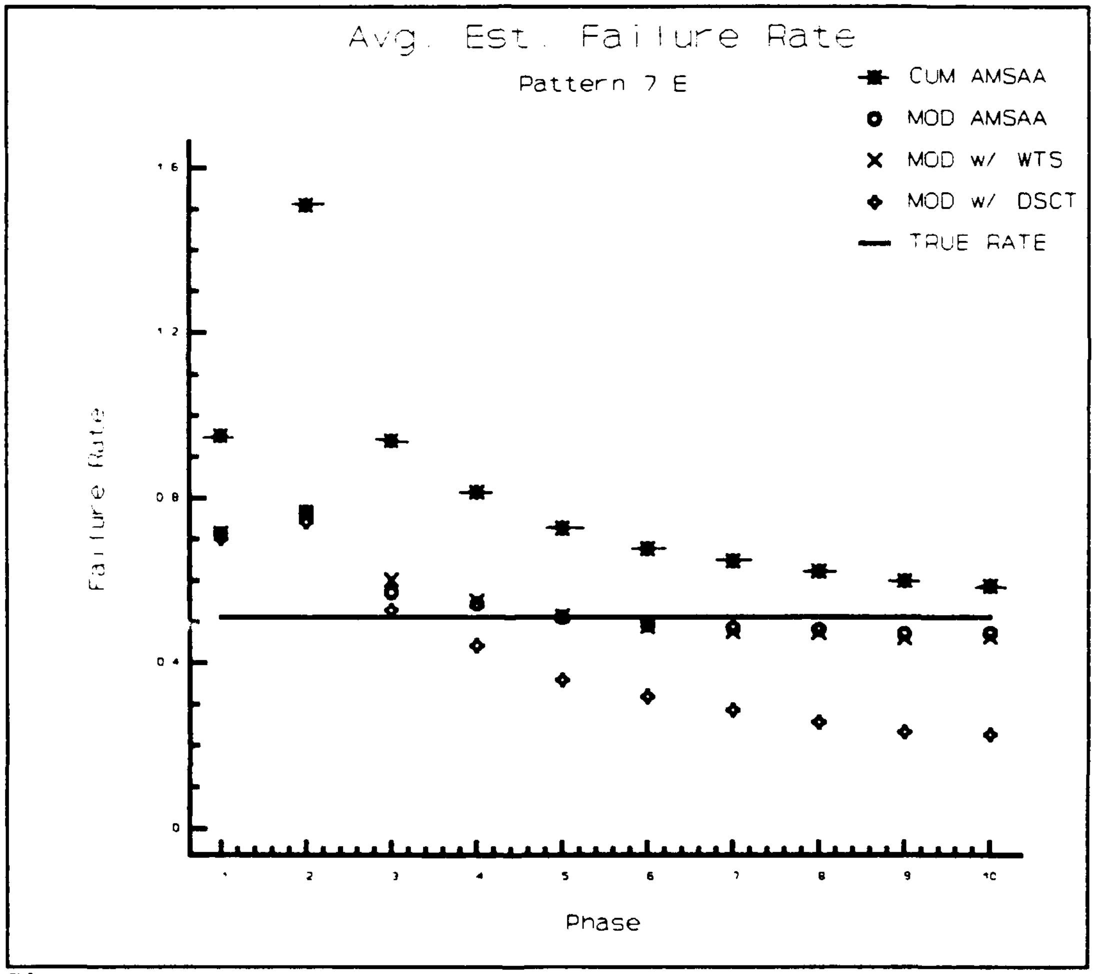


Figure 15: Pattern 7 E

The preceding examples are representative of the results for all other patterns and runs. The graphical results of all simulation runs conducted with these models can be found in Appendix C.

The Modified AMSAA model without discounting or weighting showed the best overall performance on all patterns simulated. There may be certain

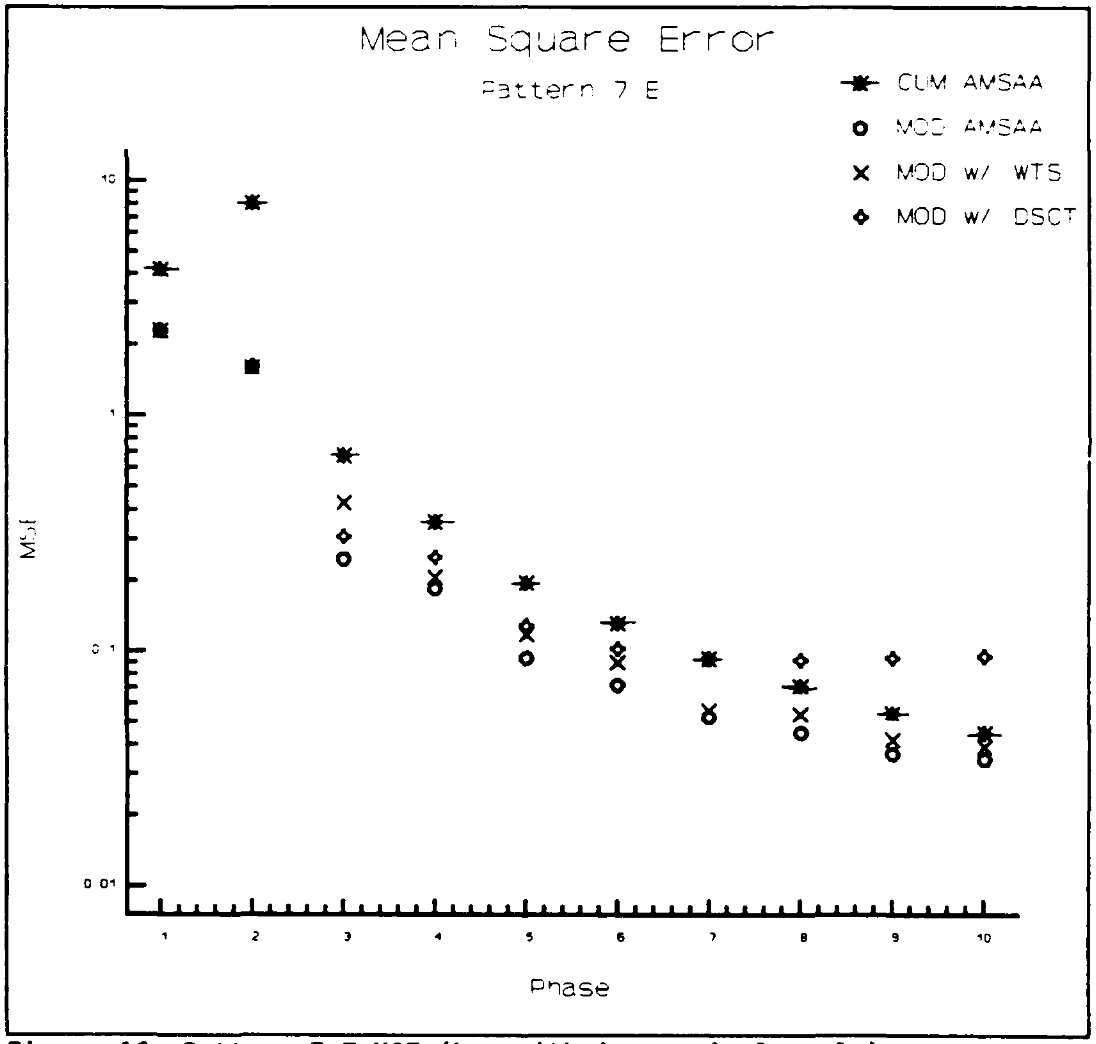


Figure 16: Pattern 7 E MSE (Logarithmic vertical scale)

reliability patterns where, given the correct parameters, the discounted or weighted versions of this model will yield a lower MSE. The reality is that, in practice, the actual failure rate is never known with certainty. This fact, coupled with the difficulty in selecting 'good' weighting parameters and computational simplicity, appear to make the Modified AMSAA model a very good choice for all applications.

VI. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

The purpose of this paprr was to evaluate the Modified AMSAA model to discover if failure discounting or regression weighting factors can improve this model. Discounting failures occurring early in a testing program should allow the early data to be used to evaluate current estimates of failure rate. If this discounting is warranted and is applied correctly, more accurate estimates of failure rate and ultimately reliability should result. Similarly the application of weighting factors was intended to improve the accuracy of the model by forcing more weight on the current phase data in the regression.

Toward this end, two methods of weighting the data were derived and evaluated. The first method allows the user to select the first phase weight and the model computes each subsequent phases weight. The other method allows the user to select the weight explicitly for each phase. Normalized weights are used in the regression equations.

The discounting method removed a certain fraction of each failure attributed to a given cause each time the required amount of test time was accumulated without that cause reoccurring. If another failure occurred due to that cause, the time since failure was reset to zero and all failures due to that cause were restored to full value. This method of discounting requires the selection of two parameters. These are the fraction to be removed, f, and the time required before removal, Treq.

B. CONCLUSIONS

The Modified AMSAA model was discovered to be superior to the AMSAA model regardless of the underlying pattern of reliability. The application of regression weighting factors or failure discounting did not significantly improve the accuracy of this model. With most patterns simulated these embellishments actually detracted from the accuracy of the Modified AMSAA model. The discounted model had a tendency to underestimate the failure rate. This is an indication that the discounting is unwarranted and probably not needed for this model.

The reason for this result could be that the Modified AMSAA model uses regression methods to estimate the failure rate from the data from each phase. This data is kept segregated by phase throughout. There may be no need to discount early failures if this early data is already handled differently than current data by the model.

The weighting methods evaluated failed to improve the Modified AMSAA model. There may be a method which optimally weights the failure data from the previous and current phases which can improve this model. The weighting methods applied in this study were not exhaustive. More analysis should be done to find better methods of weighting this model.

The Modified AMSAA model is very easy to apply to virtually any test plan. The calculations required could be done on most hand-held programmable calculators. The subroutine located in Appendix B, could be used by anyone using this model on a computer that will support Fortran.

C. RECOMMENDATIONS FOR FURTHER STUDY

The following is a list of areas which suggest further study.

- Some method optimally weighting the data should be sought so that the regression generated estimates have the lowest variance. A study of many possible weighting methods should be conducted. Ultimately a method of calculating the weights entirely from the data would be best.

- A sensitivity study should be conducted on the choice of failure discounting parameters in hopes of finding a way to optimally discount failures. Possibly a method where these parameters are calculated dynamically from the data could be found.

- Some exploration of other methods for improving the Modified AMSAA model should be studied. These would include applying jack-knifing to the regression model.

APPENDIX A

A. DERIVATION ESTIMATE OF SINGLE PHASE FAILURE RATE ESTIMATE

The Modified AMSAA model assumes an exponential failure distribution with the failure rate constant within each phase of development. The failure rate changes after the system configuration is changed, marking the transition to a new phase. Let λ_i denote the failure rate in phase i. In the test plan simulated in this thesis, testing is conducted on n items in each phase until a set number of failures, F, have been observed. After the testing is completed, failures are attributed to specific causes and changes are made in the design attempting to eliminate these causes.

The regression equations of the Modified AMSAA model require an estimate of single phase failure rate, λ_i. This estimate is used in the regression equations to estimate the parameters a and b in the model of the instantaneous failure rate,

$$\hat{\lambda}_{TTj} = (1-\hat{a})\hat{b}(TT_j)^{-\hat{a}} \qquad \text{[ref. 1: p. 3-1].}$$

In this thesis the test plan simulated tested in each phase until a predetermined number of failures had occurred. The maximum likelihood estimate for λ_j,

$$\hat{\lambda}_j = \frac{F_j}{T_j}, \text{ has expected value } E[\hat{\lambda}_j] = \frac{F_j}{F_j-1} \bullet \lambda_j, \text{ if } F_j > 1.$$

To calculate this expected value note that $2\lambda_j T_j$ has a Chi-square distribution with parameter $2F_j$. If $X = 2\lambda T$ is Chi-square(2F), then

$E[1/X] = 1/[2(F-1)]$. Using this estimate in the model produced biased results. The estimated instantaneous failure rate λ_{TTj} was consistently high.

The next estimate tried was the bias corrected maximum likelihood estimate,

$$\hat{\lambda}_j = \frac{F_j-1}{F_j} \bullet \frac{F_j}{T_j}, \text{ which has expected value } E[\hat{\lambda}_j] = \lambda_j.$$

Using this estimate the model also produced biased results. The estimated instantaneous failure rate λ_{TTj} was consistently low. Runs A through D used this estimate. The results are not included in this thesis because they were unsatisfactory.

These results prompted the search for a nearly unbiased estimate of single phase failure rate that would give unbiased results in the model. The nearly unbiased estimate chosen was

$$\hat{\lambda}_j = \frac{2F_j-1}{F_j} \bullet \frac{F_j}{T_j}, \text{ which has expected value } E[\hat{\lambda}_j] = \frac{2F_j-1}{2F_j-2} \bullet \lambda_j,$$

if $F_j > 2$. This estimate produced the results shown in this thesis for runs E through G. The variance of this estimate can be shown to be

$$\mathrm{Var}[\hat{\lambda}_j] = \frac{(2F_j-1)^2}{4(Fj-1)^2(F_j-2)} \bullet \lambda_j^2, \text{ if } F_j > 2.$$

The calculation of this variance also uses the Chi-square distribution of $2\lambda_j T_j$. If $X = 2\lambda T$ is Chi-square(2F) then $E[1/X^2] = 1/[4(F-1)(F-2)]$, and $\mathrm{Var}[1/X^2] = E[1/X^2] - (E[1/X])^2 = 1/[4(F-1)^2(F-2)]$.

B. DERIVATION OF THE MODIFIED AMSAA MODEL

The Modified AMSAA model uses exponential regression to estimate the parameters a and b which relate the amount of accumulated test time, TT to the failure rate. The model is

$$\lambda_{TTj} = (1-a)b(TT_j)^{-a} \qquad \text{[ref. 1: p. 3-1].}$$

This relationship is transformed in order to use the least squares linear regression equations to estimate a and b. If the natural log is taken of both sides of this equation, the result is

$\ln(\lambda_{TTj}) = \ln\{(1-a)b\} + (-a)\ln(TT_j) = \alpha + \beta\{\ln(TTj)\}$, if we let $\alpha = \ln\{(1-a)b\}$ and $\beta = -a$. Thus the data pairs become $(\ln(\lambda_j), \ln(TT_j))$. Applying the standard regression equations the estimates for a and b become

$$\hat{a}_j = \frac{\sum_{i=1}^{j} X_i Y_i - \overline{Y}_j \sum_{i=1}^{j} X_i}{\overline{X}_j \sum_{i=1}^{j} X_i - \sum_{i=1}^{j} X_i^2}$$

$$\hat{b}_j = \frac{1}{1-\hat{a}_j} \exp(\overline{Y}_j + \hat{a}_j \overline{X}_j)$$

where $Y_i = \ln(\hat{\lambda}_i)$ and $X = \ln(TT_i)$, $\overline{Y}_j = (Y_1+Y_2+\ldots+Y_j)/j$ and $\overline{X}_j = (X_1+X_2+\ldots+X_j)/j$.

APPENDIX B

The computer program written for this thesis is written in modular form. The program is written in six sections: the main program, the uniform pseudo-random number generator, and the four subroutines, one for each reliability growth model. The main program reads the data file, CRG.DAT, and uses the parameters contained in it to control the simulation. The simulation consists of three distinct steps. These are generation of the random variables, processing these random variables into the failure data needed by the subroutines, and finally use of the data by the four subroutines to estimate failure rates. The program also collects data on the estimates generated in order to generate statistics. These statistics, the average and mean square error are used evaluate performance of the models with respect to the "known" reliability growth pattern used to generate the data.

The subroutines could be used for any program that read the appropriate data and used to track reliability growth for an actual system that the user was interested in. It could also be used to evaluate any continuous reliability growth model provided the user programs a subroutine in such a way that it interfaces with the main program.

The names of the variables and arrays used in the main program and subroutines have been used so that they are similar to those parameters and variables in the models presented in the body of this thesis.

The remainder of this appendix is an explanation of variables and arrays used in the main program, a listing of the source code for the program and subroutines, and sample input and output files.

A. ARRAY AND VARIABLE LIST

Name	Description
NCAUSE	The number of failure causes in the simulation
NPHASE	The number of phases to be simulated
NREPS	The number of replications to simulate over
NITEMS( )	The number of items on test in each phase, indexed by phase
R( )	The number of failures per phase, indexed by phase
FCAUSE(,)	The cause of a given failure, indexed by phase and failure
CAUSE	Index for failure causes
PHASE	Index for phases of development
J	Index for failures within a phase
F(,)	The number of failures for a given cause in each phase, indexed by phase and cause
FAIL	Counter of total number of failures over all phases and causes, used to index FTT( ), for the Cumulative AMSAA model
FTT( )	Total test time accumulated by all items after each failure, indexed by failure
WTP	Weighting type pointer - Data file parameter to select weighting method (1 or 2)
T(, ,)	Exponential failure time for the first failure of n items for a given cause in each phase, indexed by phase, cause, and failure
TT(,)	The time on test for all items until the next failure occurs for any cause, indexed by phase and failure

PTT( )	Total test time accumulated by all items for each phase, indexed by phase
TSF(,)	Test time accumulated since last failure for a given cause at the end of each phase, indexed by phase and cause
LAM(,)	The failure rate (lambda) for a given cause in each phase, indexed by phase and cause
TRLAM( )	True lambda - the sum over all causes of LAM(,) for each phase, indexed by phase
LAMBR1( )	The average of the Modified AMSAA model (MODEL 1) estimate of lambda for each phase, indexed by phase
LAMBR2( )	The average over all replications of the Weighted Modified AMSAA model (MODEL 2) estimate of lambda for each phase, indexed by phase
LAMBR3( )	The average over all replications of the Discounted Modified AMSAA model (MODEL 3) estimate of lambda for each phase, indexed by phase
LAMBR4( )	The average over all replications of the Cumulative AMSAA model (MODEL 4) estimate of lambda for each phase, indexed by phase
MSE1( )	The mean square error over all replications of the Modified AMSAA model (MODEL 1) estimate of lambda for each phase, indexed by phase
MSE2( )	The mean square error over all replications of the Weighted Modified AMSAA model (MODEL 2) estimate of lambda for each phase, indexed by phase
MSE3( )	The mean square error over all replications of the Discounted Modified AMSAA model (MODEL 3) estimate of lambda for each phase, indexed by phase
MSE4( )	The mean square error over all replications of the Cumulative AMSAA model (MODEL 1) estimate of lambda for each phase, indexed by phase
FADJ( )	The adjusted number of failures for a given cause at the end of each phase, indexed by phase and cause
SEED	The seed for the random number generator function RAND(SEED)

B. LISTING OF COMPUTER PROGRAM SOURCE CODE: CRG.FOR

```
*****************************************************************
*                                                               *
*  Fortran program to Simulate Continuous failure data from    *
*  failure rates for specific causes.  The data is used to test *
*  four Reliability Growth Models and evaluate results by      *
*  comparing Mean Square Error.                                 *
*  This program will handle up to 15 phases, 10 failure causes, *
*  and 10 failures per phase.  If needed the dimensions on the  *
*  arrays could be changed to accommodate more.                 *
*                                                               *
*                                                               *
*                                                               *
*  PROGRAMMED BY :                                              *
*         LT S.L.NEGUS USN                                      *
*                                                               *
*****************************************************************


      INTEGER NCAUSE,NPHASE,NREPS,NITEMS(15),R(15),FCAUSE(15,10),
     @        CAUSE,PHASE,F(15,10),FAIL,WTP
      DOUBLE PRECISION T(15,10,10),TT(15,10),PTT(15),TSF(0:15,10)
     @    ,LAM(15,10),LAMBR1(15),LAMBR2(15),LAMBR3(15),LAMBR4(15)
     @    ,MSE1(15),MSE2(15),MSE3(15),MSE4(15),FADJ(15,15),SEED
     @    ,FTT(0:150),TRLAM(15)

      OPEN (UNIT=1,FILE='CRG.DAT')
      OPEN (UNIT=9,FILE='MSES.OUT')
      OPEN (UNIT=5,FILE='AVGS.OUT')

*  TAKE DATA FROM DATA FILE AND INITIALIZE VARIABLES

*  READ IN # CAUSES, # PHASES, # REPLICATIONS, SEED & WEIGHTING *  TYPE
POINTER FROM DATA FILE
      READ(1,*) NCAUSE,NPHASE,NREPS,SEED,WTP

*  READ IN FAILURE DISCOUNTING FRACTION AND TIME INTERVAL
      READ(1,*) FRAC,TREQ

*  READ IN FAILURE RATE FOR EACH PHASE AND CAUSE
      READ(1,*) ((LAM(PHASE,CAUSE),PHASE=1,NPHASE),CAUSE=1,NCAUSE)
*  READ IN  # ITEMS & # FAILURES PER PHASE FROM DATA FILE
      DO 1 PHASE = 1,NPHASE
         READ (1,*) NITEMS(PHASE),R(PHASE)
         DO 2 CAUSE = 1,NCAUSE
            TRLAM(PHASE) = TRLAM(PHASE)+LAM(PHASE,CAUSE)
2        CONTINUE
1     CONTINUE
```

```
*   MAIN DO LOOP TO CARRY OUT PROCEDURE NREPS TIMES

        DO 100 I=1,NREPS
        FAIL = 0
        FTT(0) = 0.
*   SIMULATE THE TIME TO FAILURE FOR THE JTH FAILURE IN EACH PHASE
*   AND FOR EACH CAUSE

          DO 10 PHASE = 1, NPHASE
            DO 15 CAUSE = 1, NCAUSE
              TSF(PHASE,CAUSE)=0.0
              DO 17 J=1,R(PHASE)
                 T(PHASE,CAUSE,J)=(-1./((NITEMS(PHASE)-J+1)*
     @                    LAM(PHASE,CAUSE)))*ALOG(RAND(SEED))
 17            CONTINUE
 15         CONTINUE

 10       CONTINUE

*   SORT THE TIMES FOR EACH CAUSE IN EACH PHASE & FAILURE TO
*   DETERMINE WHICH CAUSE FAILED FIRST AND WHEN THAT HAPPENED

          DO 20 PHASE = 1,NPHASE
            PTT(PHASE)=0.0
            DO 25 J = 1,R(PHASE)
              FCAUSE(PHASE,J) = 0
              TT(PHASE,J) = 1.0E25
              DO 30 CAUSE = 1,NCAUSE
                 IF (T(PHASE,CAUSE,J) .LT. TT(PHASE,J)) THEN
                    TT(PHASE,J) = T(PHASE,CAUSE,J)
                    FCAUSE(PHASE,J) = CAUSE
                 ENDIF
 30            CONTINUE

*   CALCULATE THE TIME SINCE LAST FAILURE FOR EACH CAUSE IN PHASE

              DO 40 CAUSE = 1,NCAUSE
                 IF (CAUSE .EQ. FCAUSE(PHASE,J)) THEN
                    TSF(PHASE,CAUSE) = 0.0
                 ELSEIF (J .EQ. 1) THEN
                    TSF(PHASE,CAUSE) = TSF(PHASE-1,CAUSE)+
     @                                 TT(PHASE,J)*(NITEMS(PHASE)-J+1)
          ELSE
                    TSF(PHASE,CAUSE) = TSF(PHASE,CAUSE)+
     @                                 TT(PHASE,J)*(NITEMS(PHASE)-J+1)
                 ENDIF

 40            CONTINUE

*   COMPUTE TOTAL TIME ON TEST FOR EACH PHASE, PTT(PHASE)
```

```
*   AND FOR EACH FAILURE, FTT(FAIL)

              PTT(PHASE)=PTT(PHASE)+TT(PHASE,J)*(NITEMS(PHASE)-J+1)
              FAIL = FAIL + 1
              FTT(FAIL)=FTT(FAIL-1)+TT(PHASE,J)*(NITEMS(PHASE)-J+1)
 25        CONTINUE

*   ADJUST FAILURES FOR FAILURE DISCOUNTING

           DO 75 K=1,PHASE
              FADJ(PHASE,K) = 0
 75        CONTINUE
           DO 50 CAUSE = 1,NCAUSE
             F(PHASE,CAUSE)=0
             DO 60 J=1,R(PHASE)
                IF (CAUSE .EQ. FCAUSE(PHASE,J)) THEN
                   F(PHASE,CAUSE) = F(PHASE,CAUSE) + 1
                ENDIF
 60          CONTINUE

             DO 70 K=1,PHASE
                FADJ(PHASE,K)=FADJ(PHASE,K)+F(K,CAUSE)
     @           *(1.- FRAC)**AINT(TSF(PHASE,CAUSE)/TREQ)
 70          CONTINUE
 50        CONTINUE
 20      CONTINUE

*   GO TO SUBROUTINES FOR MODIFIED AMSAA MODELS

          CALL MODL1(I,PTT,R,NPHASE,LAMBR1,MSE1,TRLAM)
          CALL MODL2(WTP,I,PTT,R,NPHASE,LAMBR2,MSE2,TRLAM)
          CALL MODL3(I,PTT,FADJ,NPHASE,LAMBR3,MSE3,TRLAM)
          CALL MODL4(I,FTT,R,NPHASE,LAMBR4,MSE4,TRLAM,FAIL)

100    CONTINUE

*  FORMAT AND WRITE OUTPUT TO FILE
       WRITE (5,140) NREPS
       WRITE (9,145) NREPS
       DO 120 PHASE=1,NPHASE
          LAMBR1(PHASE)=LAMBR1(PHASE)/NREPS
          MSE1(PHASE)=MSE1(PHASE)/NREPS
          LAMBR2(PHASE)=LAMBR2(PHASE)/NREPS
          MSE2(PHASE)=MSE2(PHASE)/NREPS
          LAMBR3(PHASE)=LAMBR3(PHASE)/NREPS
          MSE3(PHASE)=MSE3(PHASE)/NREPS
          LAMBR4(PHASE)=LAMBR4(PHASE)/NREPS
          MSE4(PHASE)=MSE4(PHASE)/NREPS

          WRITE (5,150) TRLAM(PHASE),LAMBR1(PHASE),LAMBR2(PHASE)
     @                ,LAMBR3(PHASE),LAMBR4(PHASE)
```

```
      WRITE (9,150) MSE1(PHASE),MSE2(PHASE),MSE3(PHASE)
     @              ,MSE4(PHASE)

120   CONTINUE

140   FORMAT ('AVERAGE OF ESTIMATES FOR',I6,' REPS'/
     @   '  TRUE LAMBDA  MODEL 1 EST  MODEL 2 EST  MODEL 3 EST',
     @ '  MODEL 4 EST')
145   FORMAT ('MEAN SQUARE ERRORS FOR',I6,' REPS'/
     @   '  MODEL 1 MSE  MODEL 2 MSE  MODEL 3 MSE  MODEL 4 MSE')
150   FORMAT (6F13.8)

      STOP
      END

*  RANDOM NUMBER GENERATOR

      FUNCTION RAND(SEED)

      DOUBLE PRECISION M,A,X,SEED
      M=2.**31-1.
      A=7.**5
      X=1. + MOD(A*SEED,M)
      RAND=X/M
      SEED=X
      RETURN
      END
```

```
******************************************************************
*                                                                *
*     SUBROUTINE #1                                              *
*                                                                *
*     FORTRAN MODEL FOR RELIABILITY IN CONTINUOUS PROCESS        *
*     PROGRAMMED BY :                                            *
*            LT S.L.NEGUS USN                                    *
*                                                                *
*     THIS MODEL IS BASED ON THE MODIFIED AMSAA MODEL.           *
*     USING LEAST SQUARES REGRESSION.                            *
*     THE CONTINUOUS FAILURE RATE IS ESTIMATED.                  *
*     NO DISCOUNTING OR REGRESSION WEIGHTS ARE USED.             *
*                                                                *
*                                                                *
******************************************************************

*  SUBROUTINE PROGRAM TO ESTIMATE LAMBDA USING MODIFIED AMSAA MODEL

      SUBROUTINE MODL1(I,T,F,NPHASE,LAMBAR,MSE,LAM)

      INTEGER F(15)
      DOUBLE PRECISION T(15),TT,X,Y,SUMXY,SUMY,XBAR,
     @  YBAR,SUMX,SUMX2,AHAT,BHAT,LAMHAT
     @  ,LAMBAR(15),MSE(15),LAM(15)


********************************************
*   MAIN DO LOOP                           *
*                                          *
*   ITERATIONS ON DEVELOPMENT PHASE, K     *
*                                          *
********************************************

      TT=0.
      SUMXY=0.
      SUMY=0.
      SUMX=0.
      SUMX2=0.


      DO 200 K=1,NPHASE
         IF (F(K) .LE. 1) THEN
            Y = ALOG(.5/T(K))
         ELSE
            Y = ALOG((2.0*F(K)-1.0)/(2.0*T(K)))
         ENDIF
         TT=TT+T(K)
         X=ALOG(TT)
         SUMXY=SUMXY+X*Y
         SUMY=SUMY+Y
```

```
      SUMX=SUMX+X
      SUMX2=SUMX2+X*X
      XBAR=SUMX/K
      YBAR=SUMY/K

      IF (K .GT. 1)  THEN
         AHAT=(SUMXY-YBAR*SUMX)/(XBAR*SUMX
   @        -SUMX2)
         BHAT=(1./(1.-AHAT))*EXP(YBAR+AHAT*XBAR)
         LAMHAT=(1-AHAT)*BHAT*TT**(-AHAT)
      ELSEIF (F(K) .GT. 1) THEN
         LAMHAT=(2.0*F(K)-1.0)/(2.0*T(K))
      ELSE
         LAMHAT=.5/T(K)
      ENDIF

      LAMBAR(K)=LAMBAR(K)+LAMHAT
      MSE(K) = MSE(K) + (LAMHAT-LAM(K))**2

200   CONTINUE

210   FORMAT(3F6.1)
220   FORMAT(I3,5F10.4)
230   FORMAT(A3,5A10)

      END
```

```
*****************************************************************
*                                                               *
*     SUBROUTINE #2                                             *
*                                                               *
*     FORTRAN MODEL FOR RELIABILITY IN CONTINUOUS PROCESS       *
*     PROGRAMMED BY :                                           *
*            LT S.L.NEGUS USN                                   *
*                                                               *
*     THIS MODEL IS BASED ON THE MODIFIED AMSAA MODEL           *
*     USING WEIGHTED LEAST SQUARES REGRESSION.                  *
*     THE CONTINUOUS FAILURE RATE IS ESTIMATED.                 *
*     REGRESSION WEIGHTS ARE USED.                              *
*                                                               *
*                                                               *
*****************************************************************

*  SUBROUTINE PROGRAM TO ESTIMATE LAMBDA USING MODIFIED AMSAA MODEL

      SUBROUTINE MODL2(WTP,I,T,F,NPHASE,LAMBAR,MSE,LAM)

      INTEGER F(15),N(15),WTP
      DOUBLE PRECISION T(15),TT(0:15),X(15),Y(15),SUMXY(15)
     @ ,SUMY(0:15),XBAR(15),YBAR(15),SUMX(0:15),SUMX2(15),AHAT(15)
     @ ,BHAT(15),LAMBAR(15),MSE(15),LAM(15),W(15),SUMW(0:15),SUMLAM
     @  ,LAMHAT(15)

      IF (I .EQ. 1) OPEN (UNIT=3,FILE='WEIGHTS.DAT')

********************************************
*   MAIN DO LOOP                           *
*                                          *
*   ITERATIONS ON DEVELOPMENT PHASE, K     *
*                                          *
********************************************

      TT(0)=0
      SUMY(0)=0
      SUMX(0)=0
      SUMW(0)=0


      SUMLAM = 0.
      DO 300 K=1,NPHASE


*  FIND THE WEIGHT FOR THIS PHASE
          IF (WTP .EQ. 1) THEN
             IF (I .EQ. 1) THEN
*  READ IN WEIGHT
                READ(3,*) W(K)
```

```
            ENDIF
         ELSE

*  COMPUTE WEIGHTS USING FRAC**(1/K) FORMULA
            IF ((I .EQ. 1).AND.(K .EQ. 1)) THEN
*  READ IN FRACTION
               READ(3,*) FRAC
            ENDIF
            W(K) = FRAC**(1./K)
         ENDIF
         SUMW(K)=SUMW(K-1)+W(K)

         IF (F(K) .LE. 1) THEN
            Y(K)=ALOG(.5/T(J))
         ELSE
            Y(K)=ALOG((2.0*F(K)-1.0)/(2.0*T(K)))
         ENDIF
         TT(K)=TT(K-1)+T(K)
         X(K)=ALOG(TT(K))
         SUMY(K)=SUMY(K-1)+Y(K)*W(K)
         SUMX(K)=SUMX(K-1)+X(K)*W(K)
         XBAR(K)=SUMX(K)/SUMW(K)
         YBAR(K)=SUMY(K)/SUMW(K)
         SUMXY(K) = 0.
         SUMX2(K) = 0.

         DO 350 J=1,K
            SUMXY(K)=SUMXY(K)+W(J)*(X(J)-XBAR(K))*Y(J)
            SUMX2(K)=SUMX2(K)+W(J)*(X(J)-XBAR(K))**2
350      CONTINUE


         IF (K .GT. 1)  THEN
             AHAT(K)=(-1.0)*SUMXY(K)/SUMX2(K)
             BHAT(K)=(1./(1.-AHAT(K)))*EXP(YBAR(K)+AHAT(K)*XBAR(K))
             LAMHAT(K)=(1-AHAT(K))*BHAT(K)*TT(K)**(-AHAT(K))
         ELSEIF (F(K) .GT. 1) THEN
             LAMHAT(K)=(2.0*F(K)-1.0)/(2.0*T(K))
         ELSE
             LAMHAT(K)=.5/T(K)
         ENDIF

         LAMBAR(K)=LAMBAR(K)+LAMHAT(K)
         MSE(K) = MSE(K) + (LAMHAT(K)-LAM(K))**2

300   CONTINUE

310   FORMAT(3F6.1)
320   FORMAT(I2,5F10.4)
330   FORMAT(A2,5A10)
      END
```

```
*********************************************************************
*                                                                   *
*     SUBROUTINE #3                                                 *
*                                                                   *
*     FORTRAN MODEL FOR RELIABILITY IN CONTINUOUS PROCESS           *
*     PROGRAMMED BY :                                               *
*            LT S.L.NEGUS USN                                       *
*                                                                   *
*     THIS MODEL IS BASED ON THE MODIFIED AMSAA MODEL.              *
*     USING LEAST SQUARES REGRESSION.                               *
*     THE CONTINUOUS FAILURE RATE IS ESTIMATED.                     *
*     FAILURE DISCOUNTING IS USED.                                  *
*                                                                   *
*                                                                   *
*********************************************************************

*  SUBROUTINE PROGRAM TO ESTIMATE LAMBDA USING MODIFIED AMSAA MODEL

      SUBROUTINE MODL3(I,T,F,NPHASE,LAMBAR,MSE,LAM)

      INTEGER N(15)
      DOUBLE PRECISION T(15),TT(0:15),X(15),Y(15),SUMXY(15),
     @ SUMY(15),XBAR(15),YBAR(15),SUMX(15),SUMX2(15),AHAT(15),
     @ BHAT(15),LAMBAR(15),MSE(15),LAM(15),F(15,15),LAMHAT(15)

********************************************
*   MAIN DO LOOP                           *
*                                          *
*   ITERATIONS ON DEVELOPMENT PHASE, K     *
*                                          *
********************************************

      TT(0)=0

      DO 200 K=1,NPHASE
         SUMXY(K)=0
         SUMY(K)=0
         SUMX(K)=0
         SUMX2(K)=0
         TT(K)=TT(K-1)+T(K)

         DO 250 J=1,K
            IF (F(K,J) .LE. 1.5) THEN
               Y(J)=ALOG(.5/T(J))
            ELSE
               Y(K)=ALOG((2.0*F(K,J)-1.0)/(2.0*T(J)))
            ENDIF
            X(K)=ALOG(TT(K))
            SUMXY(K)=SUMXY(K)+X(J)*Y(J)
```

```
          SUMY(K)=SUMY(K)+Y(J)
          SUMX(K)=SUMX(K)+X(J)
          SUMX2(K)=SUMX2(K)+X(J)*X(J)
250     CONTINUE

        XBAR(K)=SUMX(K)/K
        YBAR(K)=SUMY(K)/K
        IF (K .GT. 1)  THEN
          AHAT(K)=(SUMXY(K)-YBAR(K)*SUMX(K))/(XBAR(K)*SUMX(K)
     @      -SUMX2(K))
          BHAT(K)=(1./(1.-AHAT(K)))*EXP(YBAR(K)+AHAT(K)*XBAR(K))

          LAMHAT(K)=(1-AHAT(K))*BHAT(K)*TT(K)**(-AHAT(K))
        ELSE IF (F(K,K) .GT. 1.) THEN
          LAMHAT(K)=(2.0*F(K,K)-1.0)/(2.0*T(K))
        ELSE
          LAMHAT(K)= .5 / T(K)
        ENDIF

        LAMBAR(K)=LAMBAR(K)+LAMHAT(K)
        MSE(K) = MSE(K) + (LAMHAT(K)-LAM(K))**2

200   CONTINUE

210   FORMAT(3F6.1)
220   FORMAT(I2,5F10.4)
230   FORMAT(A2,5A10)

      END
```

```
*******************************************************************
*                                                                 *
*     SUBROUTINE #4                                               *
*                                                                 *
*     FORTRAN MODEL FOR RELIABILITY IN CONTINUOUS PROCESS         *
*     PROGRAMMED BY :                                             *
*            LT S.L.NEGUS USN                                     *
*                                                                 *
*     THIS MODEL IS BASED ON THE AMSAA MODEL.                     *
*     THE CONTINUOUS FAILURE RATE IS ESTIMATED.                   *
*                                                                 *
*                                                                 *
*******************************************************************

*  SUBROUTINE PROGRAM TO ESTIMATE LAMBDA USING AMSAA MODEL

      SUBROUTINE MODL4(I,X,R,NPHASE,LAMBAR,MSE,TRLAM,NFAIL)

      INTEGER R(15),NPHASE,NFAIL,I,J,K,N
      DOUBLE PRECISION X(0:150),LAMBAR(15),MSE(15),TRLAM(15),SUM
     @   ,AHAT(15),BHAT(15),LAMHAT(15)

      N=0
      DO 110 K=1,NPHASE
         N=N+R(K)
         SUM = 0.0

         IF (N .GT. 2) THEN
*  CALCULATE SUMMATION FOR ESTIMATES
           DO 100 J=1,N-1
             IF (X(N)/X(J).GT.0.) THEN
               SUM = SUM + ALOG(X(N)/X(J))
             ELSE
               PRINT *,'ERROR IN SUBROUTINE 4; NEG TIME AT REP ',I
               STOP
             ENDIF
100        CONTINUE

*  CALCULATE ESTIMATES OF PARAMETERS AND FAILURE RATE
           IF (SUM.NE.0.) THEN
             AHAT(K) = N/SUM
           ELSE
             PRINT *,'ERROR IN SUBROUTINE 4; BAD SUM REP ',I
             STOP
           ENDIF
           BHAT(K) = N/(X(N)**AHAT(K))
           LAMHAT(K) = AHAT(K)*BHAT(K)*X(N)**(AHAT(K)-1.0)
         ELSE
           LAMHAT (K) = N/X(N)
         ENDIF
         LAMBAR(K) = LAMBAR(K) + LAMHAT(K)
```

```
         MSE(K) = MSE(K) +(LAMHAT(K)-TRLAM(K))**2

110   CONTINUE

*  ERROR CHECK:  IS FINAL N = NFAIL?

      IF (N .NE. NFAIL) THEN
         PRINT *,'ERROR IN SUBROUTINE 4; DISAGREEMENT IN
     @ # FAILURES',I
         STOP
      ENDIF

      END
```

C. SAMPLE USER DATA FILE: CRG.DAT

```
5,10,500,12345.,1  NUMBER OF CAUSES, PHASES, REPS, SEED, WT SCHEME
.5,3.    DISCOUNTING FRACTION, TIME INTERVAL
 .0101  .2485  .1054  .0408  .1054  .1054  .0408  .1054  .2485  .0101
 .2485  .1054  .0408  .1054  .0101  .0101  .1054  .0408  .1054  .2485
 .1054  .0408  .1054  .0101  .2485  .2485  .0101  .1054  .0408  .1054
 .0408  .1054  .0101  .2485  .1054  .1054  .2485  .0101  .1054  .0408
 .1054  .0101  .2485  .1054  .0408  .0408  .1054  .2485  .0101  .1054
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 1
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 2
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 3
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 4
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 5
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 6
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 7
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 8
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 9
5,2     NUMBER OF ITEMS AND FAILURES IN PHASE 10
```

D. SAMPLE OUTPUT FILES: AVGS.OUT AND MSES.OUT

1. AVGS.OUT

AVERAGE OF ESTIMATES FOR 500 REPS

TRUE LAMBDA	MODEL 1 EST	MODEL 2 EST	MODEL 3 EST	MODEL 4 EST
.51020000	.71328117	.71328117	.70052859	.95104156
.51020000	.76562159	.76562159	.74092911	1.50951350
.51020000	.56986163	.57546390	.52658969	.93848540
.51020000	.54309383	.54768820	.44106557	.81369015
.51020000	.50977294	.51543531	.35891453	.72585658
.51020000	.49092691	.49542132	.31900548	.67593184
.51020000	.48351571	.48732093	.28569272	.64586001
.51020000	.47925587	.55408753	.25654757	.62174413
.51020000	.46936838	.50655576	.23365031	.59856596
.51020000	.47131996	.50018151	.22722248	.58609344

2. MSES.OUT

MEAN SQUARE ERRORS FOR 500 REPS

MODEL 1 MSE	MODEL 2 MSE	MODEL 3 MSE	MODEL 4 MSE
2.28307676	2.28307676	2.29055386	4.17982537
1.59749806	1.59749806	1.61366470	8.03775319
.24587519	.26528818	.30665339	.67145094
.18450393	.18848388	.25013123	.35297324
.09295514	.09939919	.12774927	.19431540
.07139884	.07607480	.10168462	.13055892
.05210290	.05293369	.09214781	.09232873
.04455537	.21215101	.09099763	.07024485
.03619510	.07225663	.09299745	.05374019
.03441349	.05769722	.09465450	.04438188

APPENDIX C

This appendix contains the graphical presentation of the results from all simulation runs conducted with the models presented and developed in this thesis. The graphs are identical in format to those presented in Chapter V. For completeness, those graphs presented in chapter 5 will be repeated here with all the other graphs.

These graphs are organized according to the runs and the reliability growth patterns within those runs. Each of these Patterns was evaluated for four sets of regression weights and four sets of discounting parameters as summarized in tables XII and XIII respectively.

Table XII: REGRESSION WEIGHTS USED

Run	1	2	3	4	5	6	7	8	9	10
E	.050	.224	.368	.473	.549	.607	.652	.688	.717	.741
F	.250	.500	.620	.707	.758	.794	.820	.841	.857	.871
G	.500	.707	.794	.841	.871	.891	.905	.917	.925	.933
H	.100	.100	.110	.120	.130	.140	.150	.700	.800	.900

Notes:
1. E, F, and G used weighting method 1, $w_i^* = f^{1/i}$, $f = w_1^*$.
2. H used weighting method 2, user selected weights.
3. All w_i^* above are divided by the sum of weights when used in the regression equations to ensure that weights always sum to one.

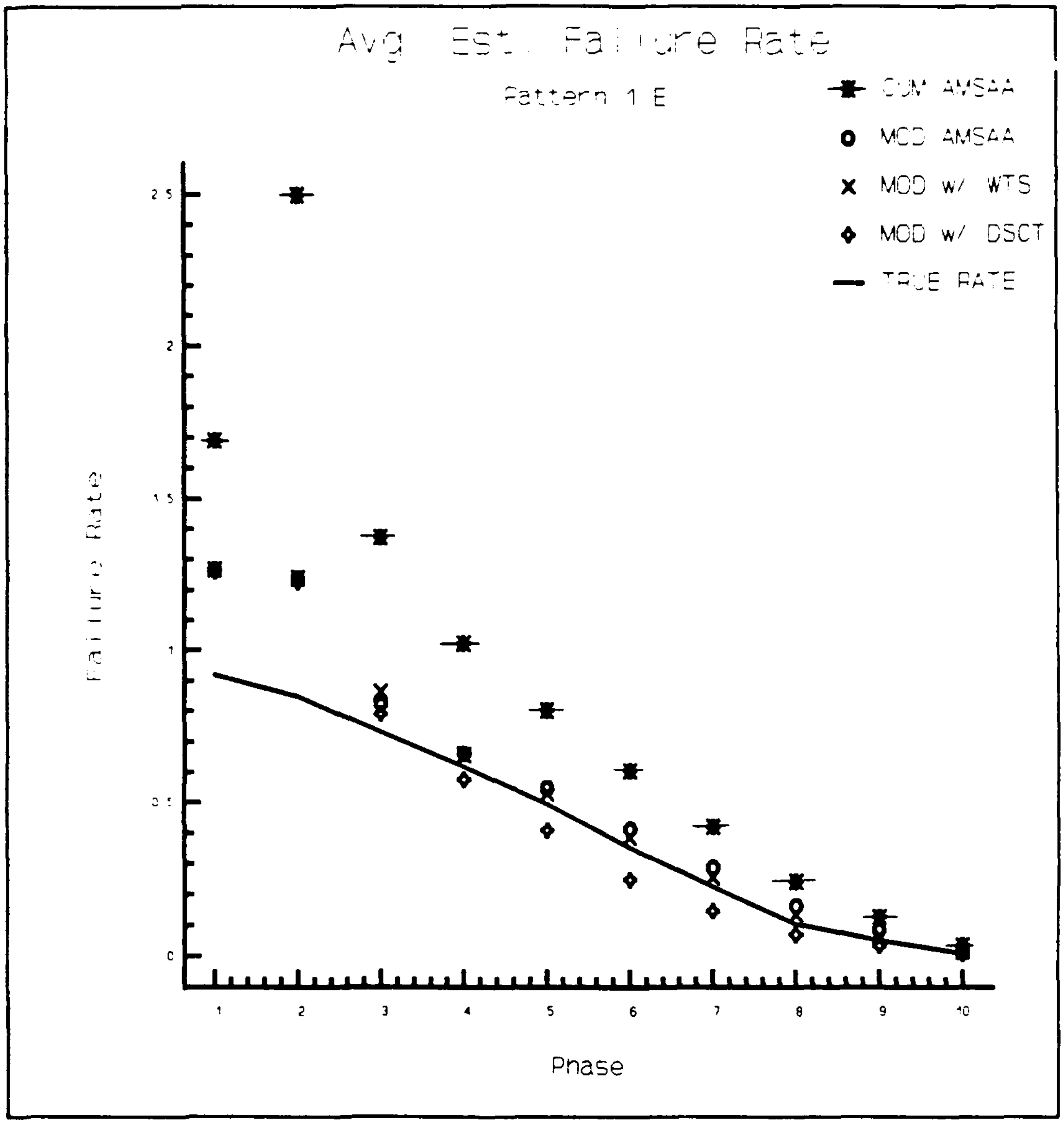
Table XIII: DISCOUNTING PARAMETERS USED

Run	Fraction	Time Required Between Discounting
E	.50	3
F	.50	15
G	.25	3
H	.25	15

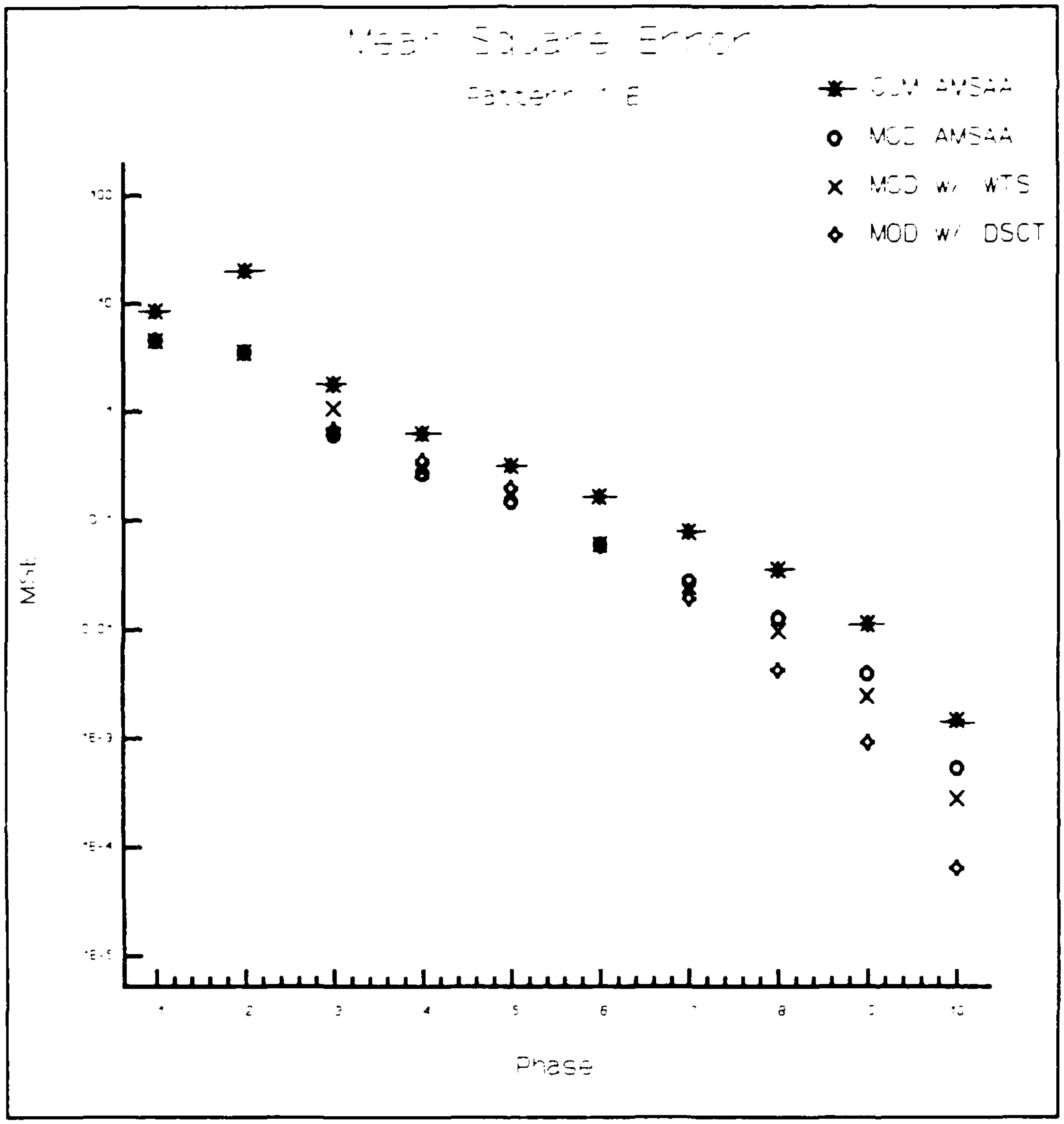
The patterns are those introduced in Chapter IV. These patterns are referred to by the numerical designator . The numerals are listed on the figures and are summarized below:

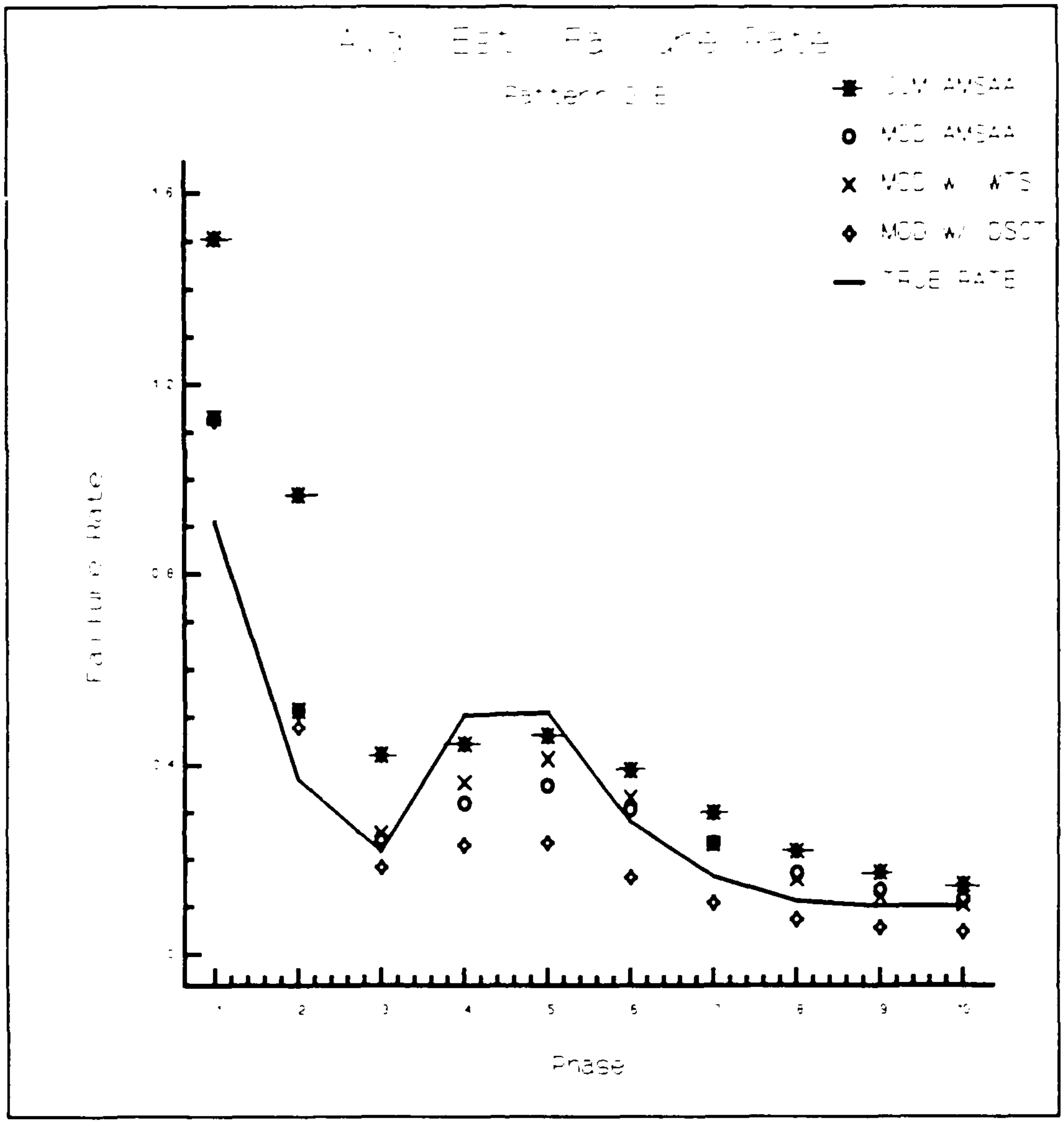
- Pattern 1 - Convexly increasing reliability
- Pattern 2 - Reliability increasing rapidly, then decreasing then increasing again
- Pattern 3 - Reliability increasing rapidly, then constant, then increasing again
- Pattern 4 - Rapidly increasing to high reliability
- Pattern 5 - Rapidly increasing to moderately high reliability
- Pattern 6 - Constant moderately high reliability
- Pattern 7 - Constant moderate reliability
- Pattern 8 - Constant low reliability

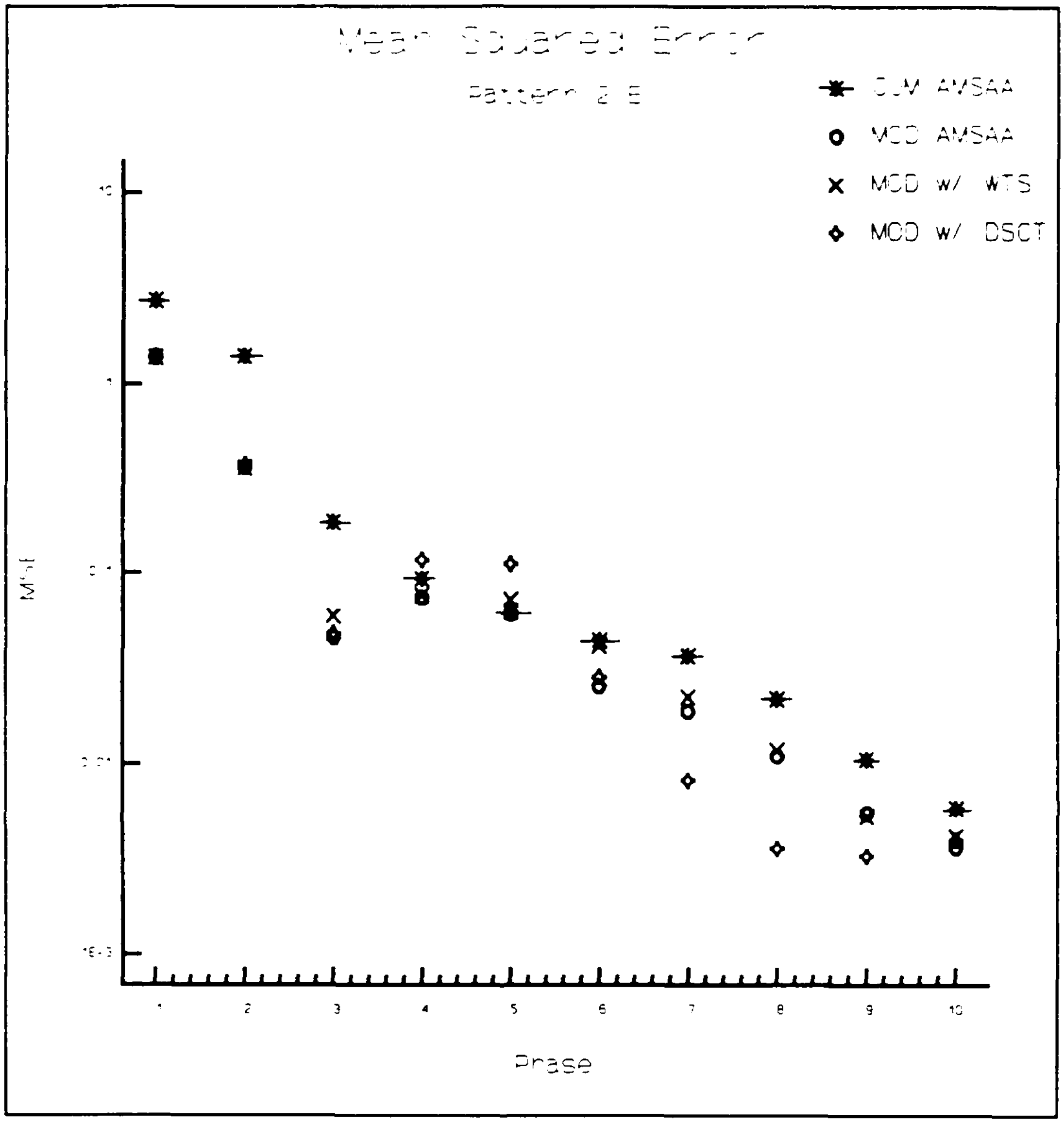
Once again increasing reliability is synonymous with decreasing failure rate.

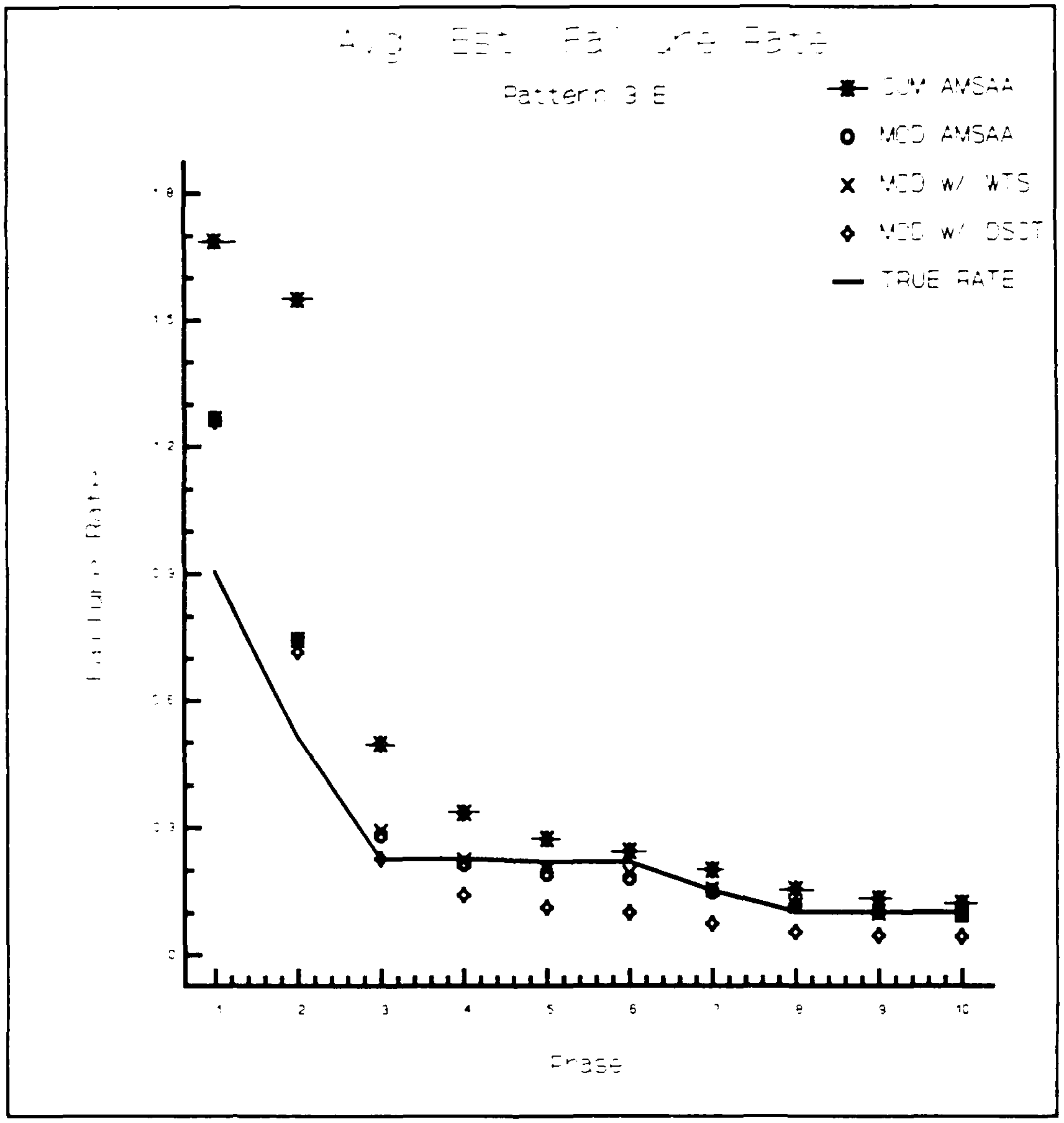


Avg. Est. Failure Rate
Pattern 1 E
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase

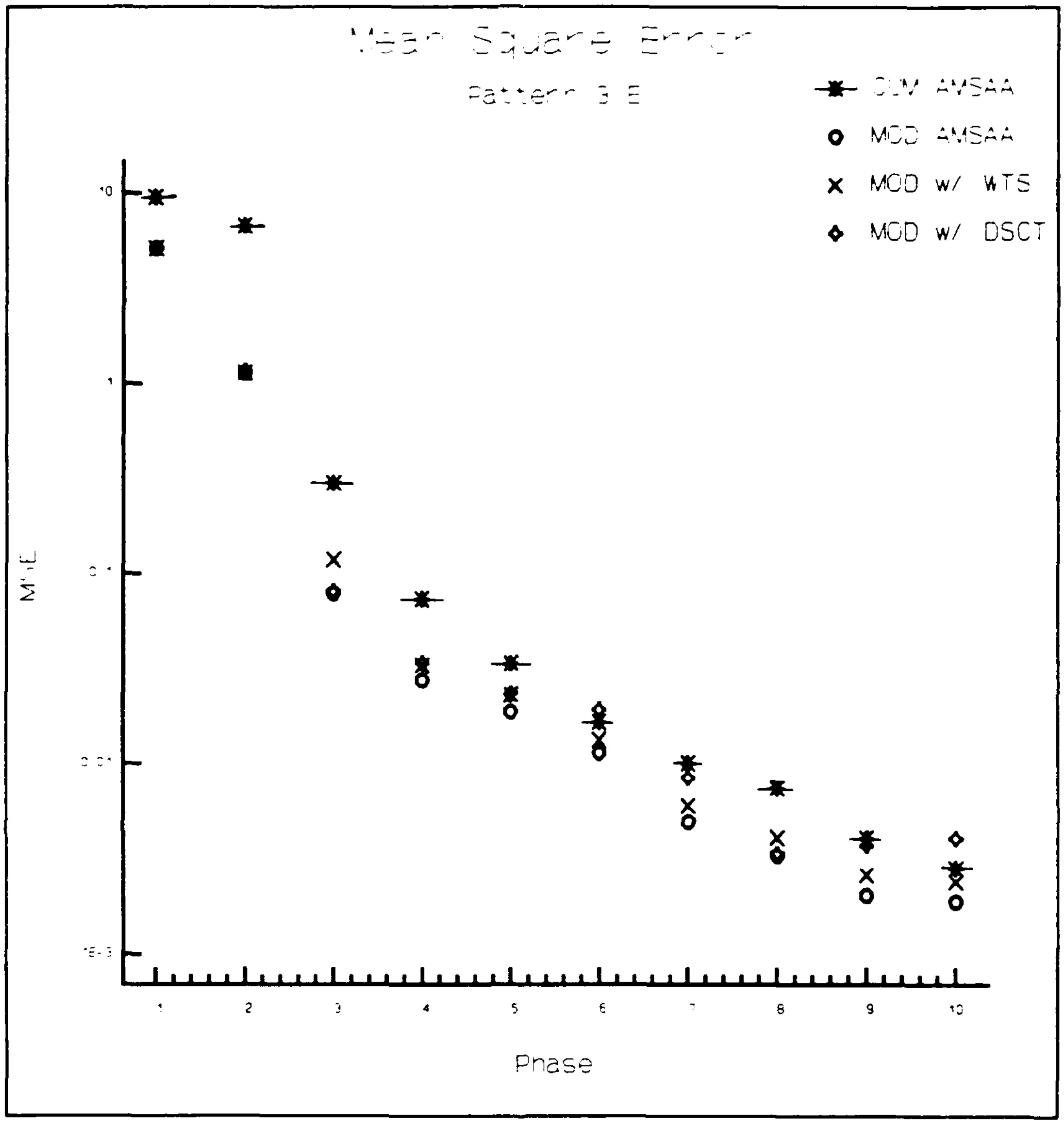

MOD w/ DSCT
Phase

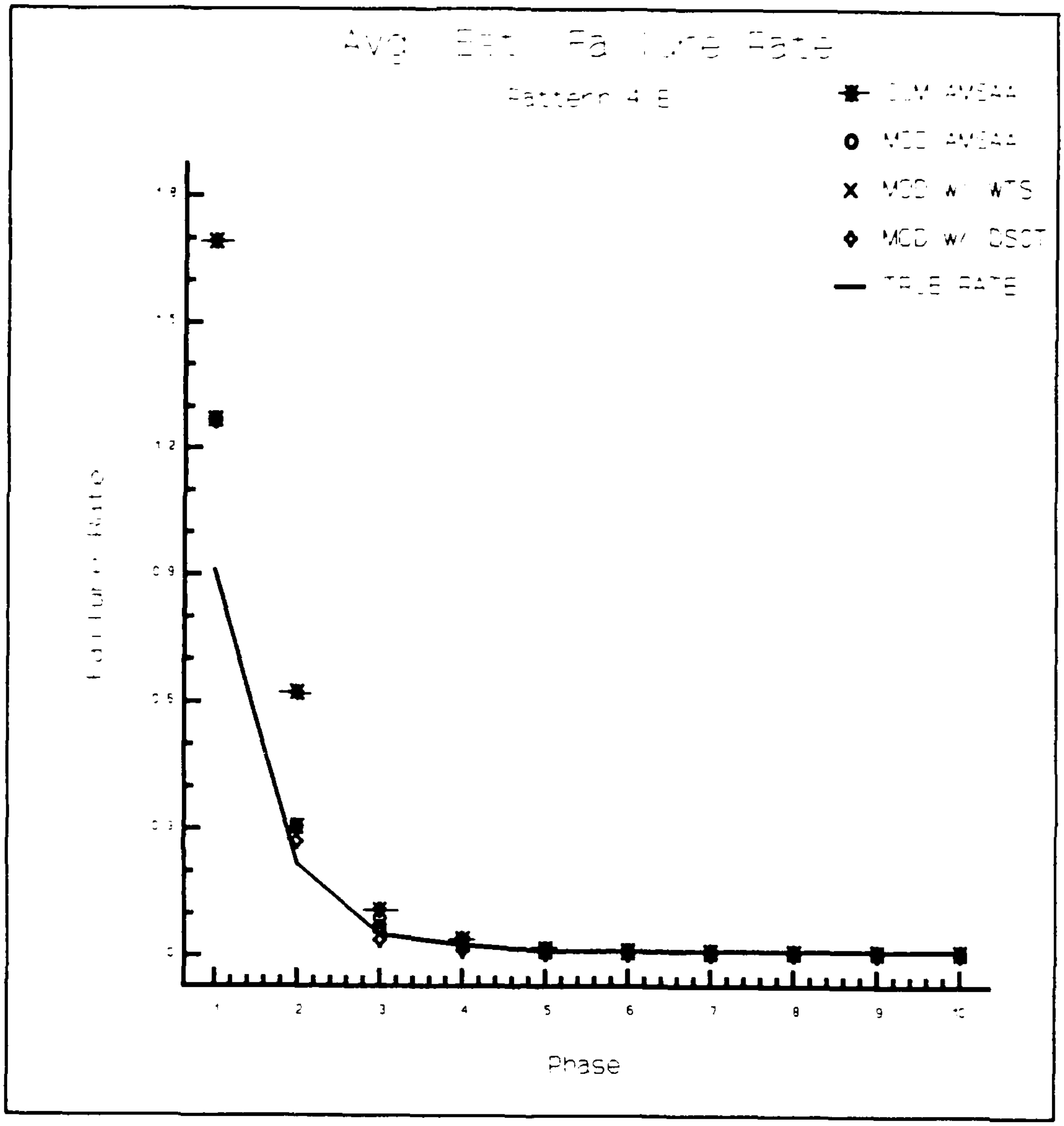


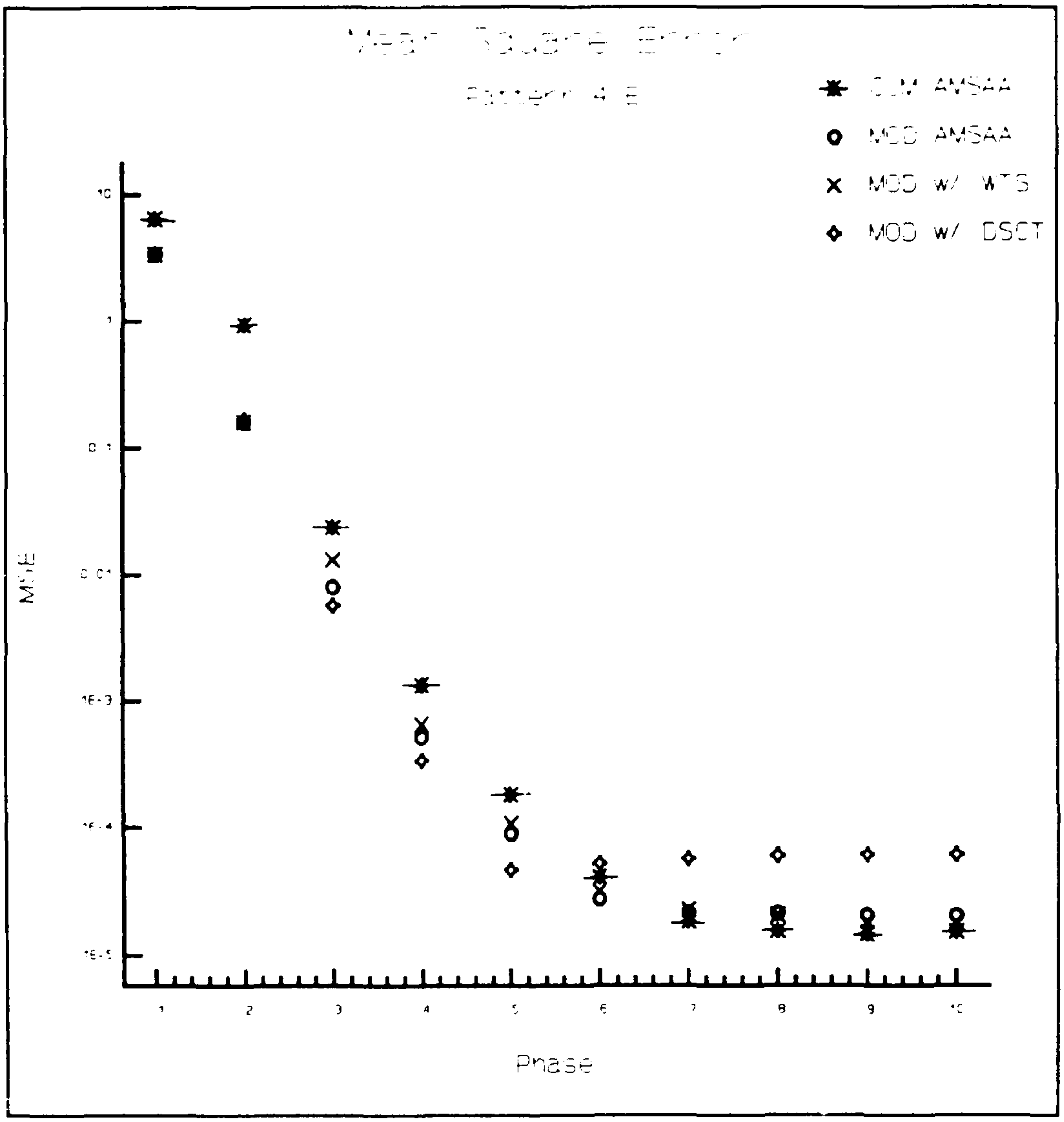

Mean Squared Error
Pattern 2 E
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
Phase

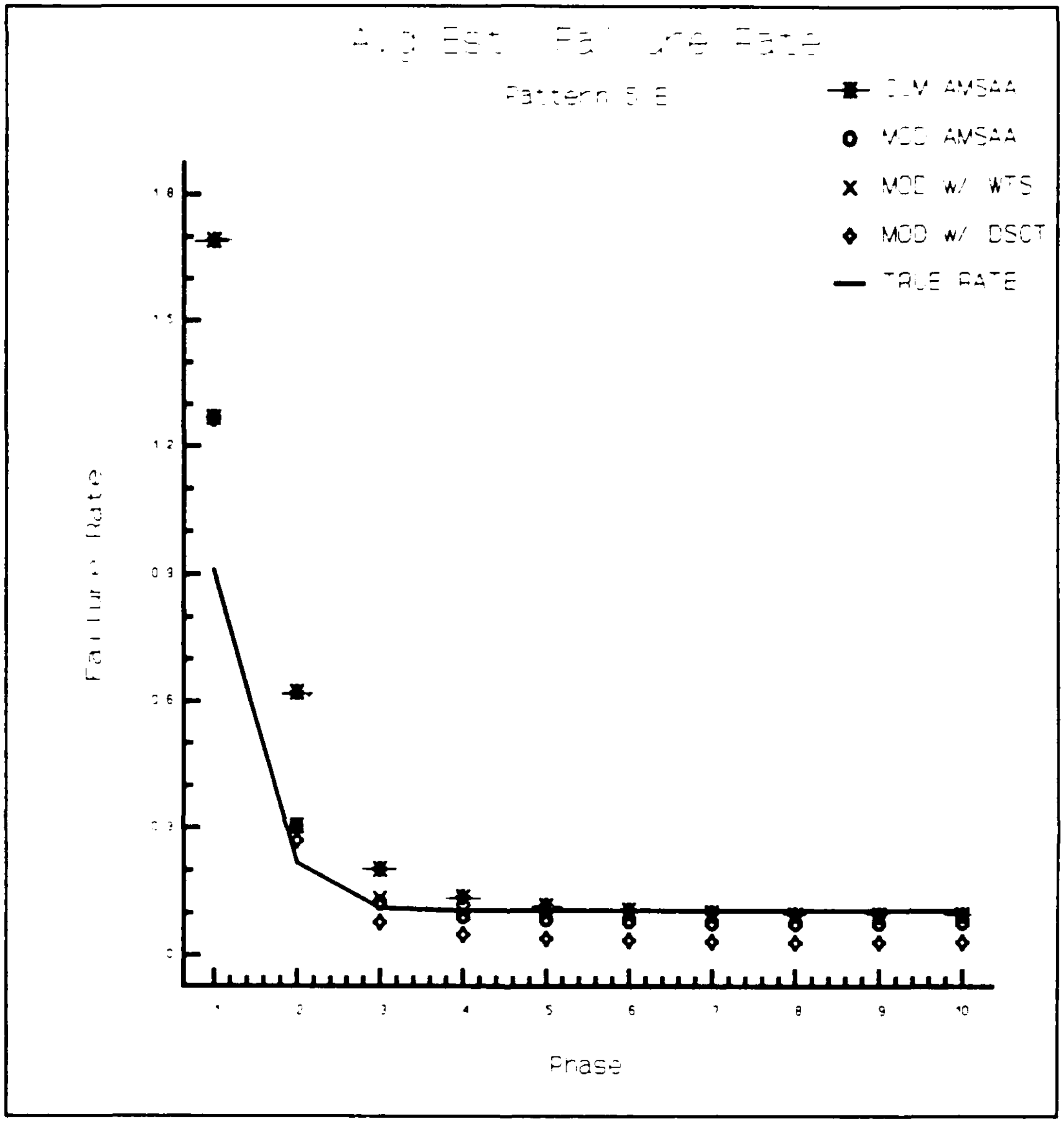


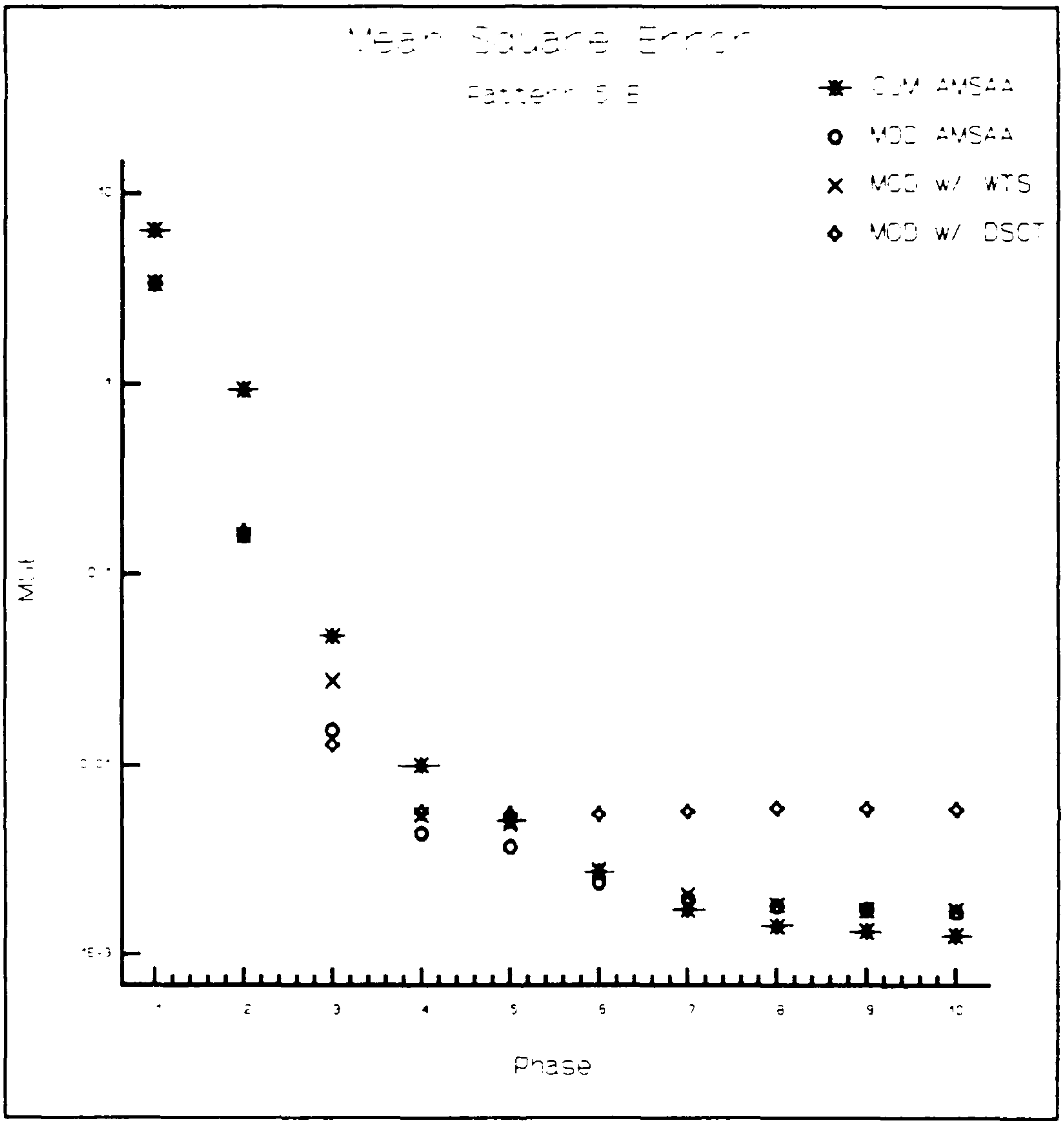
Pattern 3 E
AMSAA
MOD AMSAA
TRUE RATE
1
2
3
4
5
6
8
9
10


Mean Square Error
Pattern 3 E
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
Phase
1
2
3
4
5
6
7
8
9
10

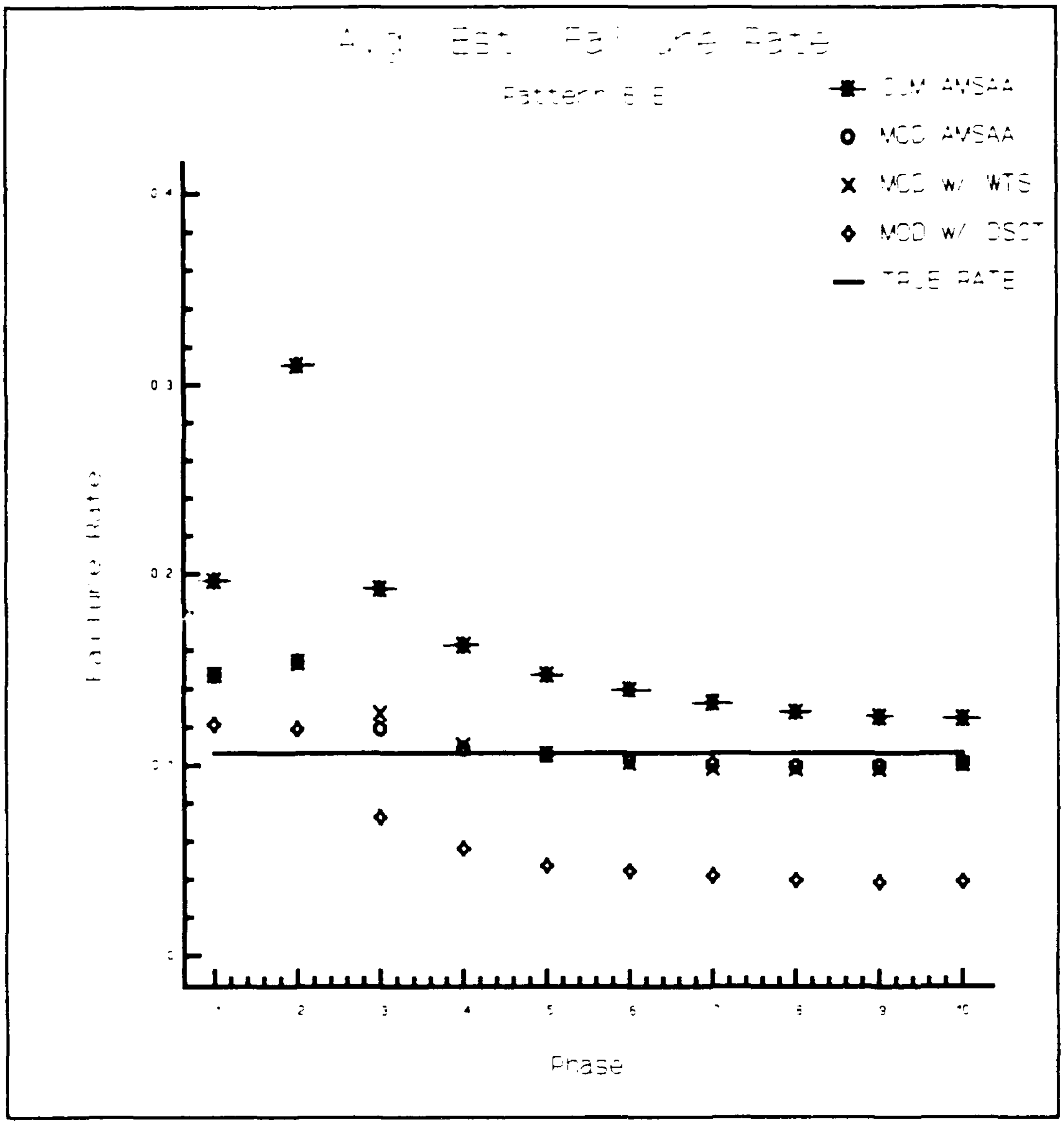

Pattern 4 E
MCD w/ WTS
MCD w/ DSCT
2
3
4
5
6
7
8
9
Phase

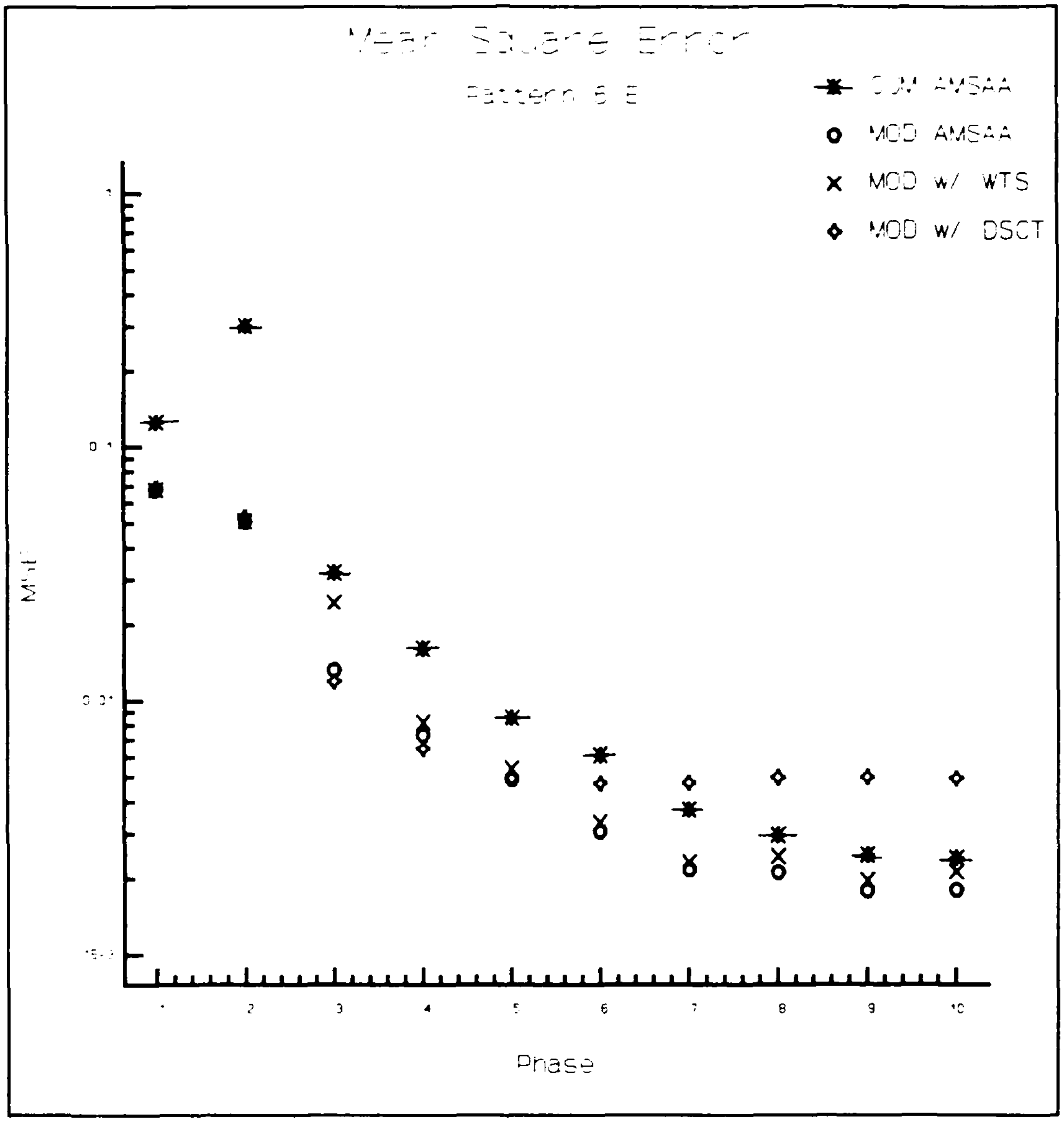

AMSAA
MOD AMSAA
MOD w/ DSCT
1
2
3
4
6
8
9
Phase


Pattern 5 E
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
1.8
1.2
0.9
0.6
0
4
5
6
8
9
10
Phase

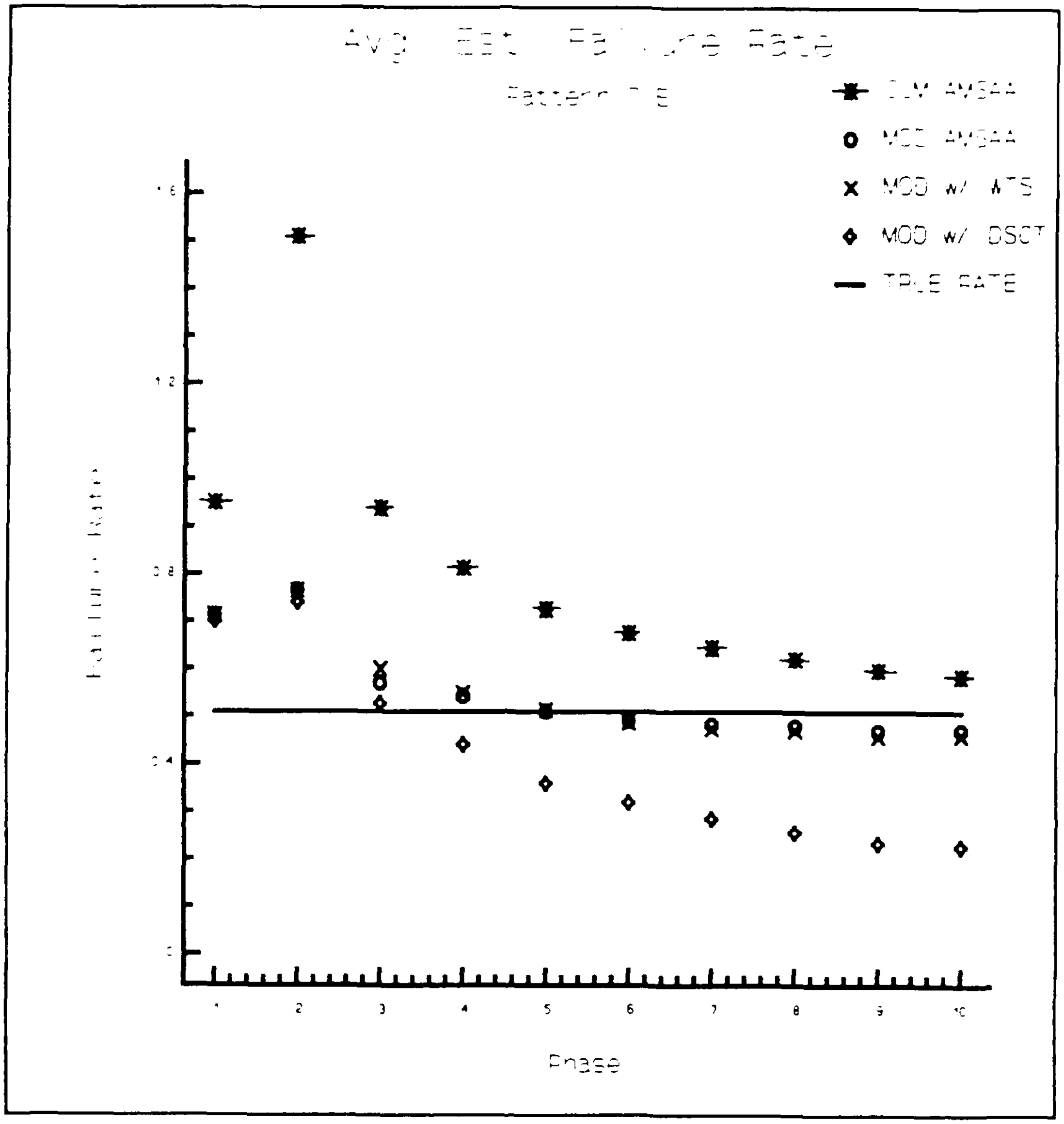


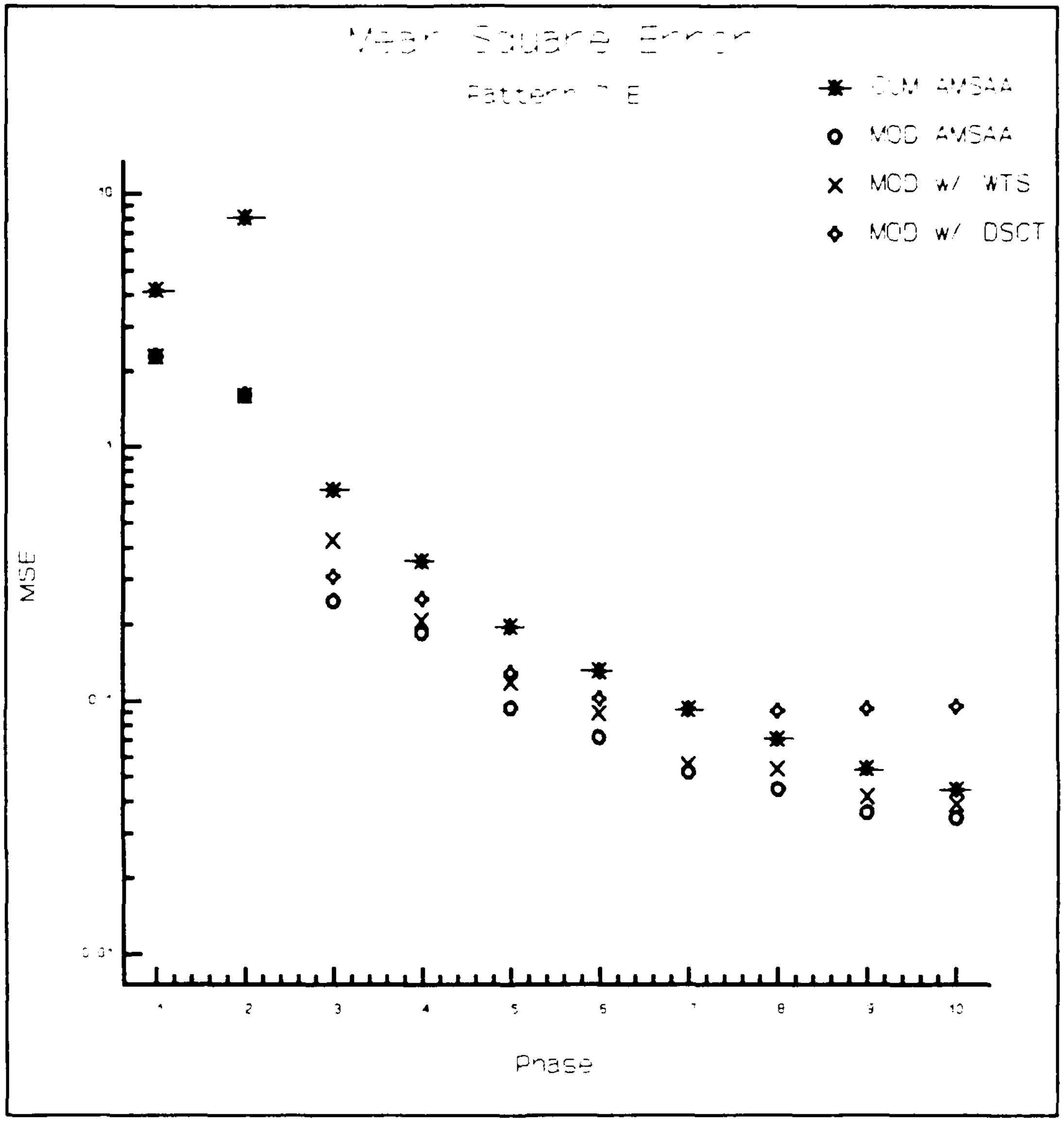
Mean Square Error
Pattern 5 E
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
Phase


Avg Est Failure Rate
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
0.4
0.3
0.2
Phase

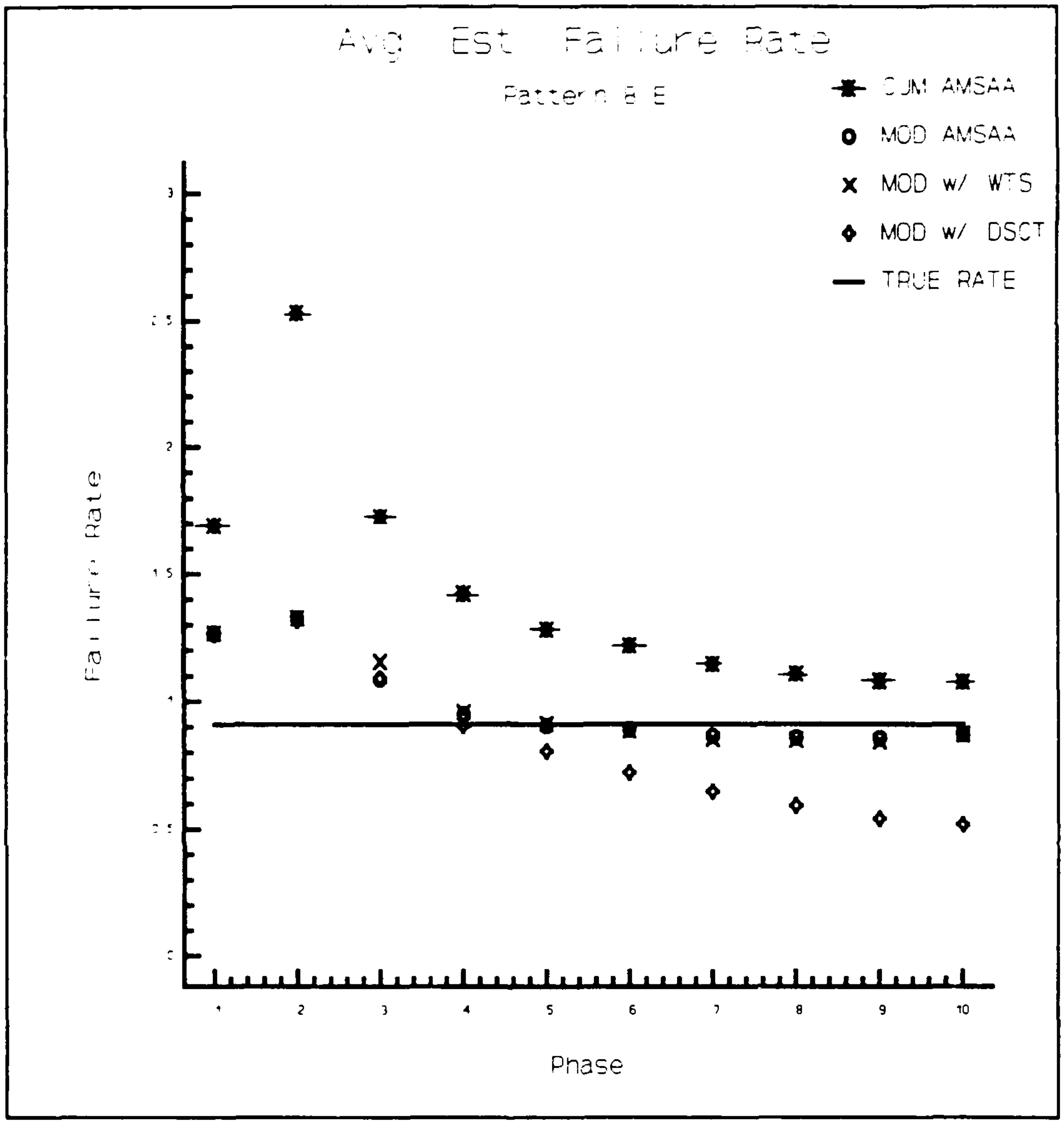


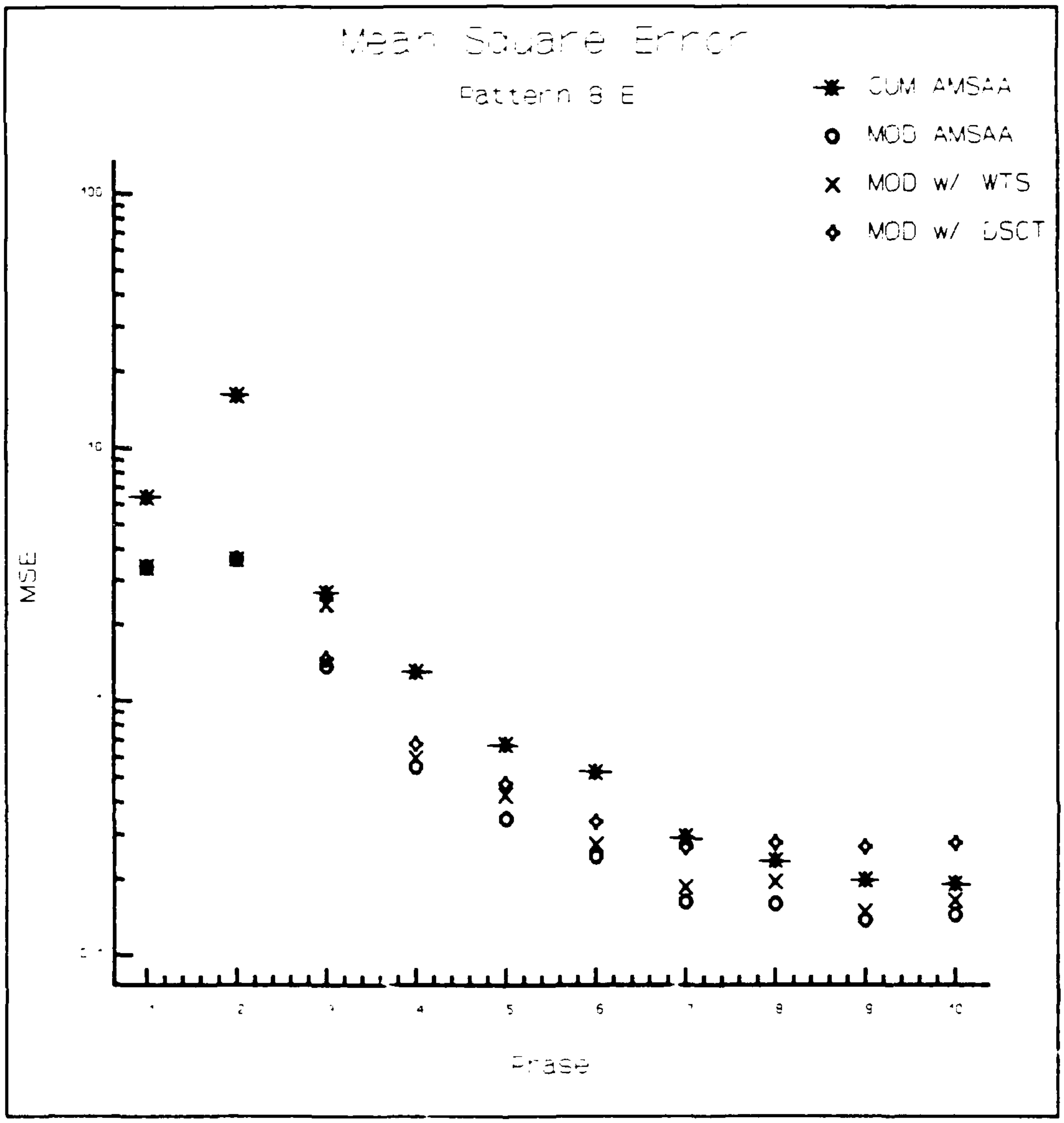
Mean Square Error
Pattern 6 E
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
1
0.1
0.01
2
3
4
6
8
9
Phase

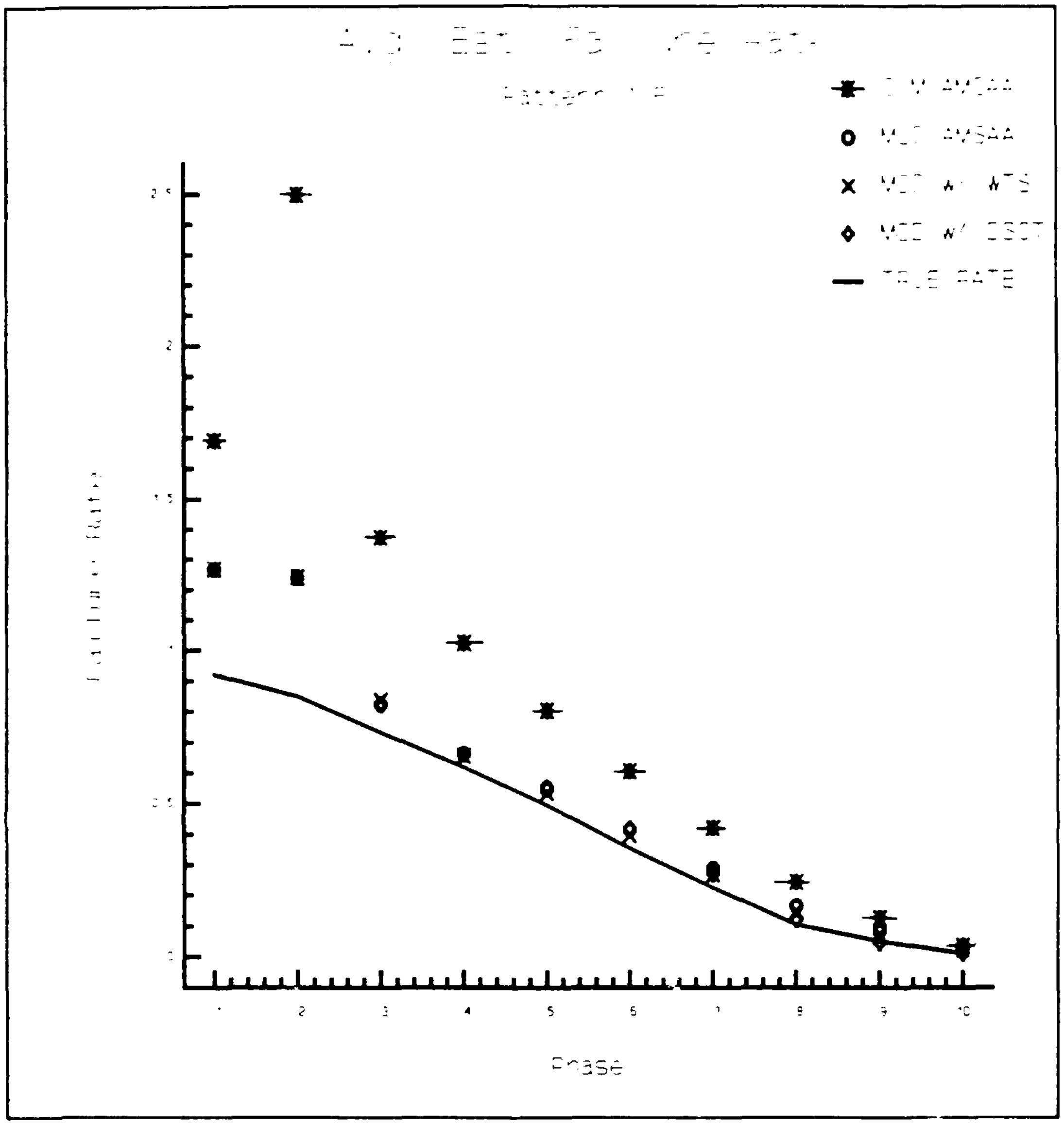

MOD w/ DSCT
Phase

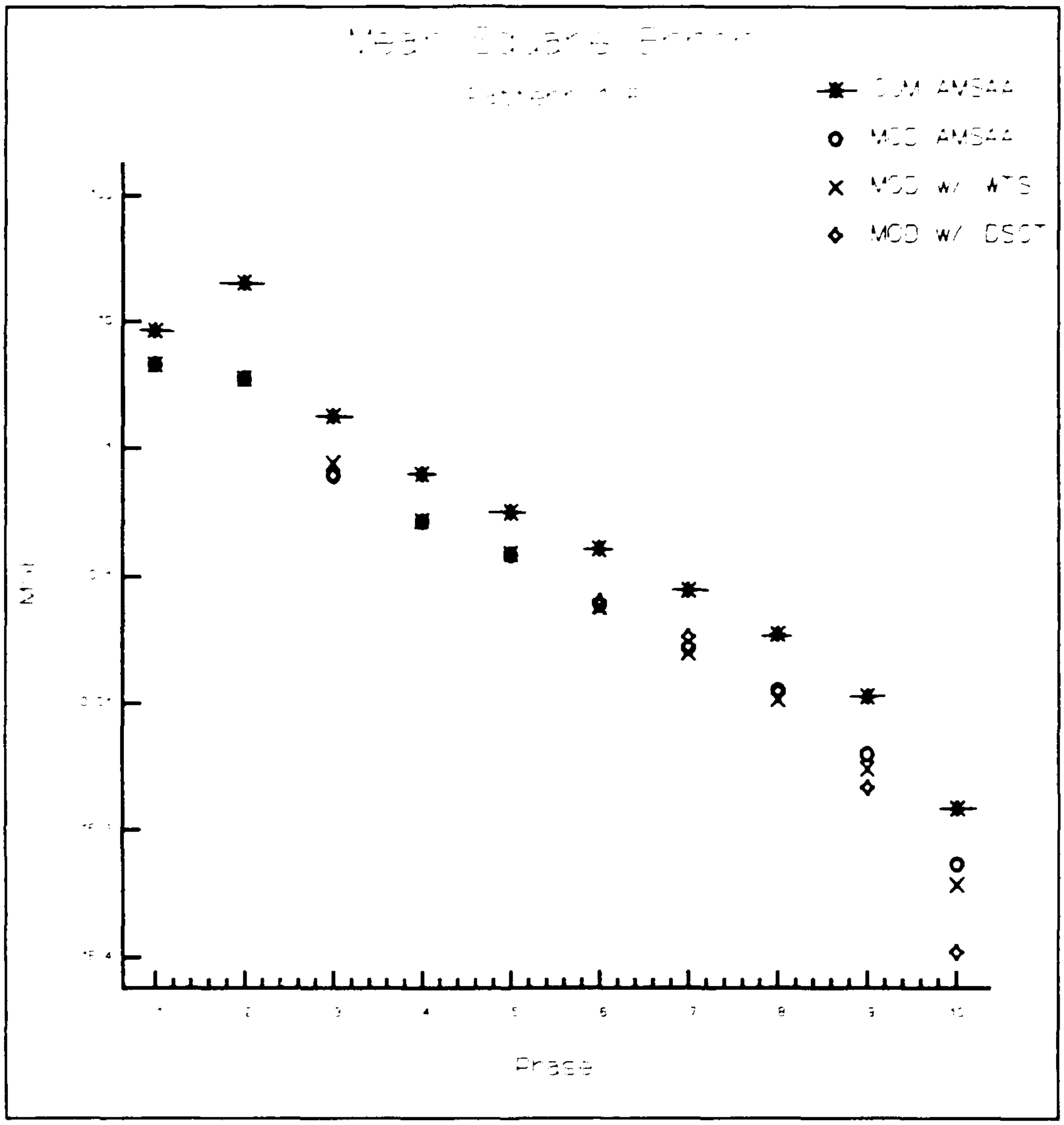


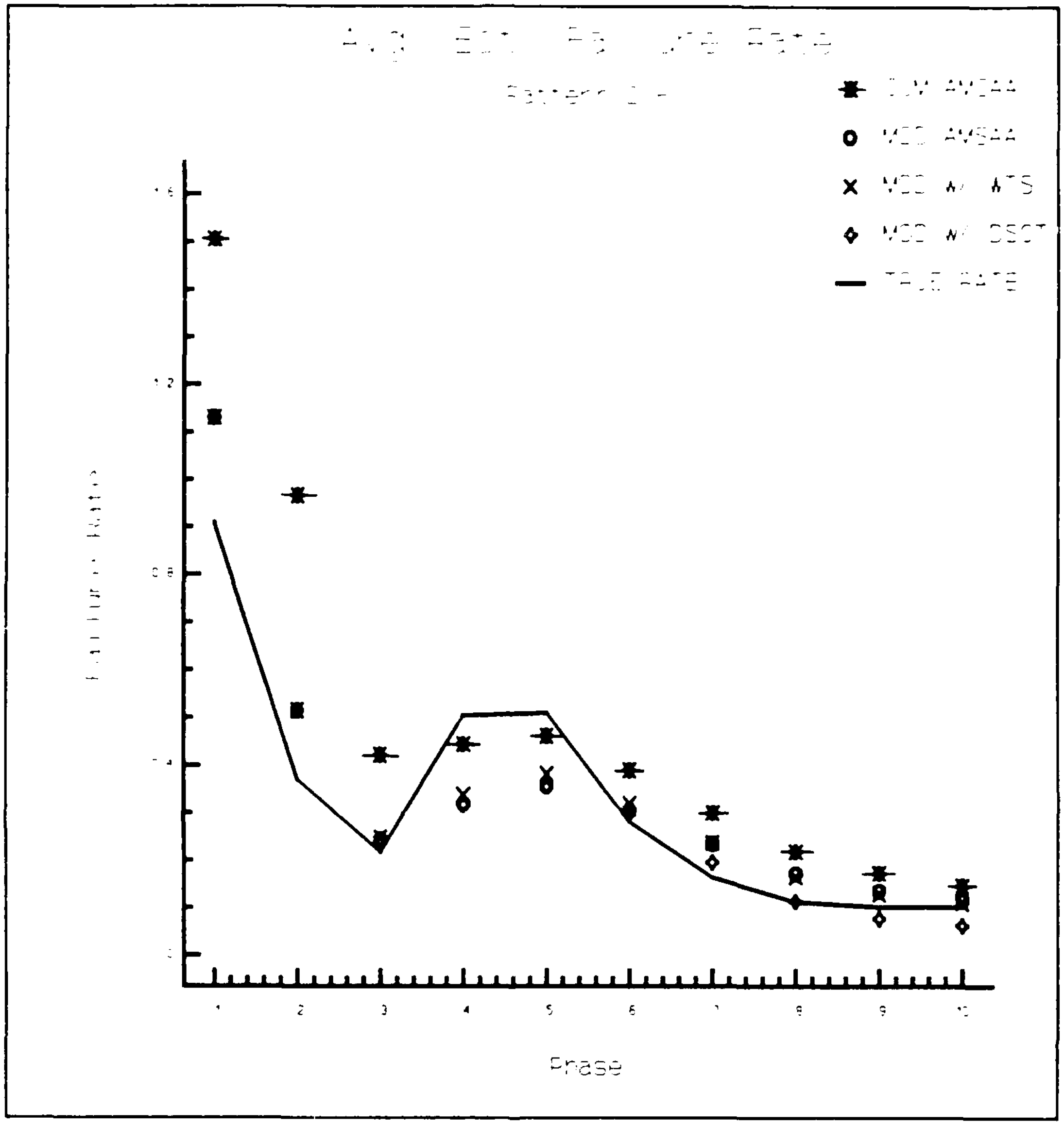
Mean Square Error
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
2
3
4
5
6
8
Phase

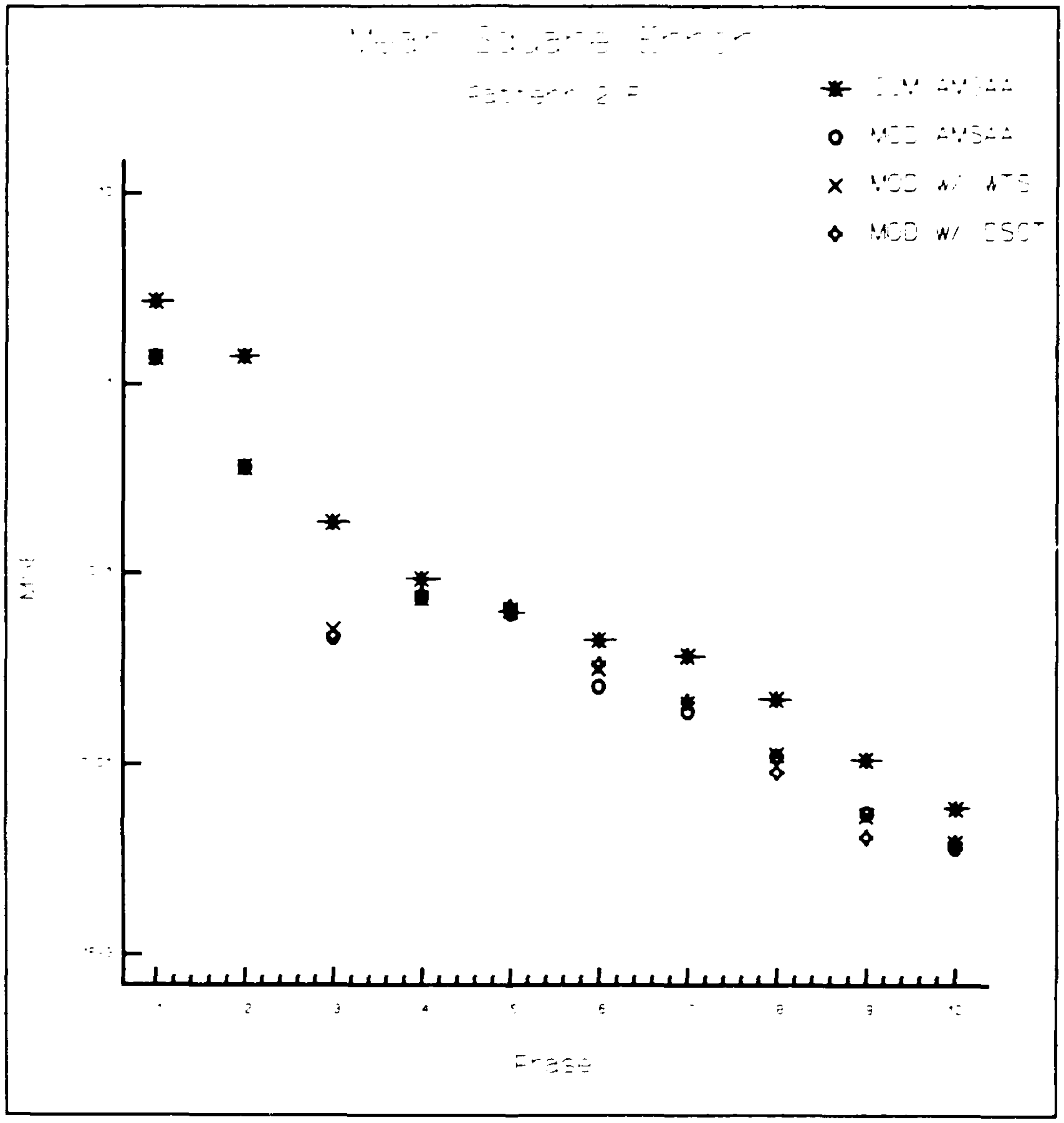

Avg. Est. Failure Rate
Pattern 8 E
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase
1
2
3
4
5
6
7
8
9
10

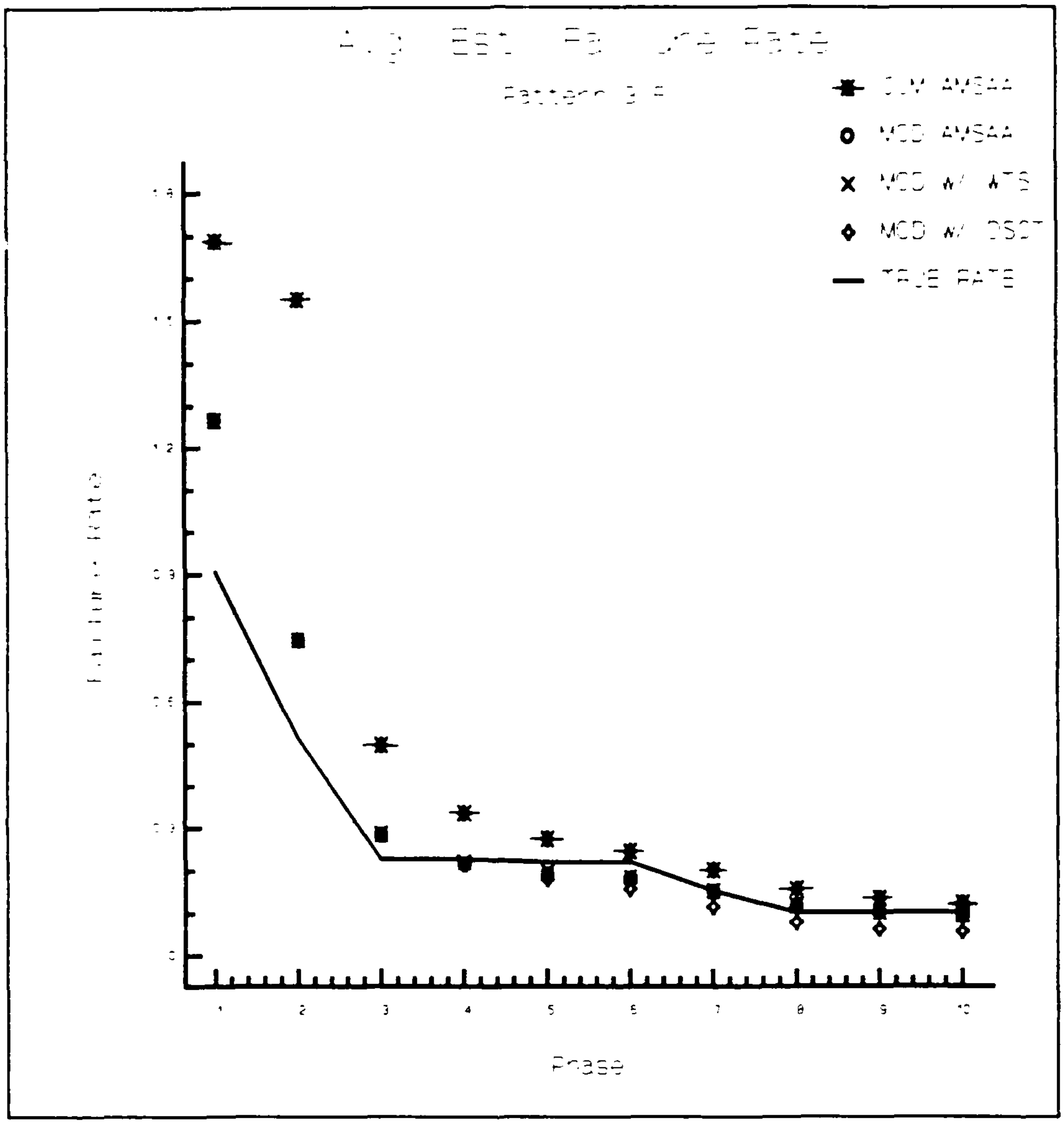

Mean Square Error
Pattern 8 E
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
100
10
Phase
2
4
5
6
8
9

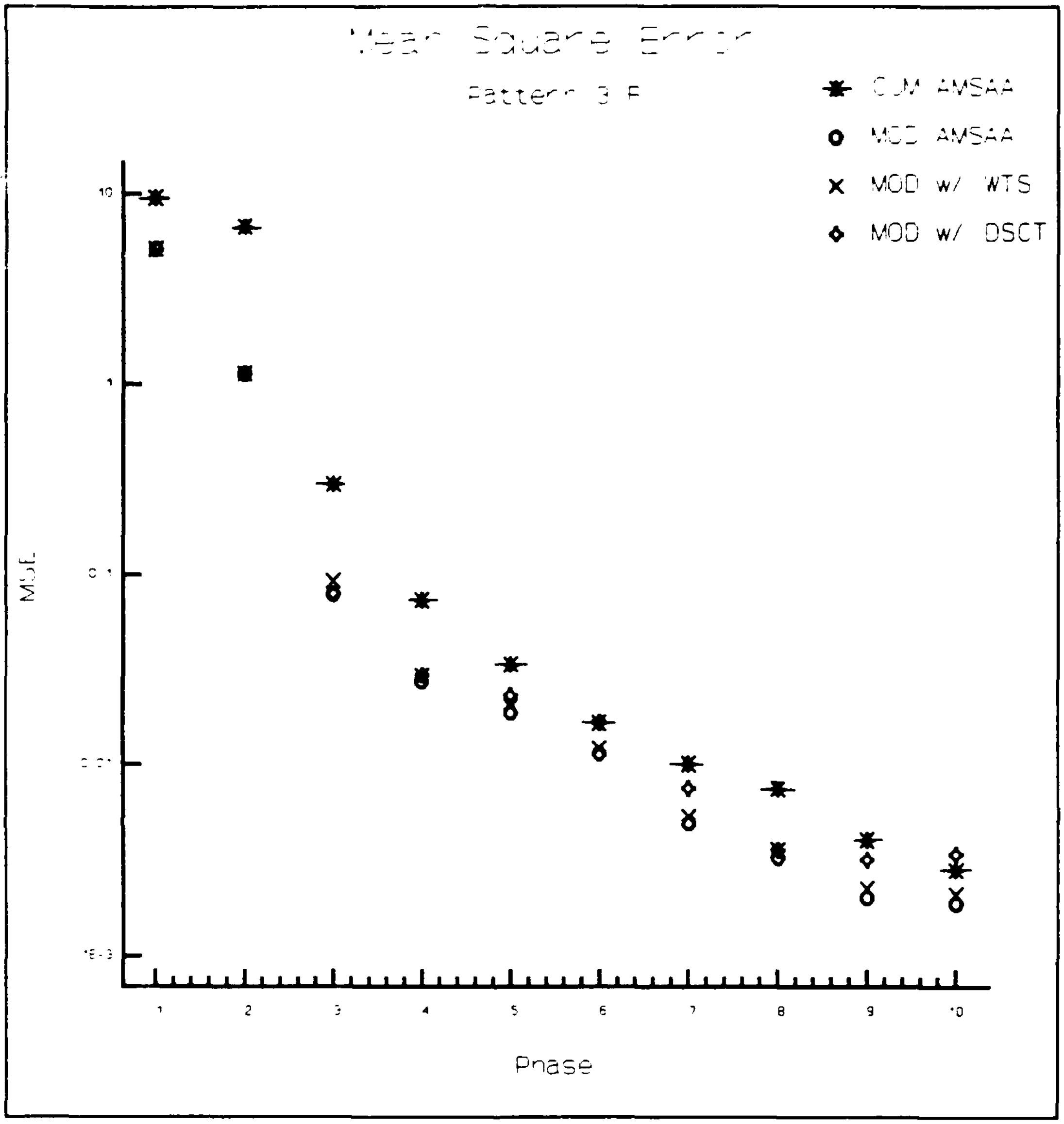

Phase

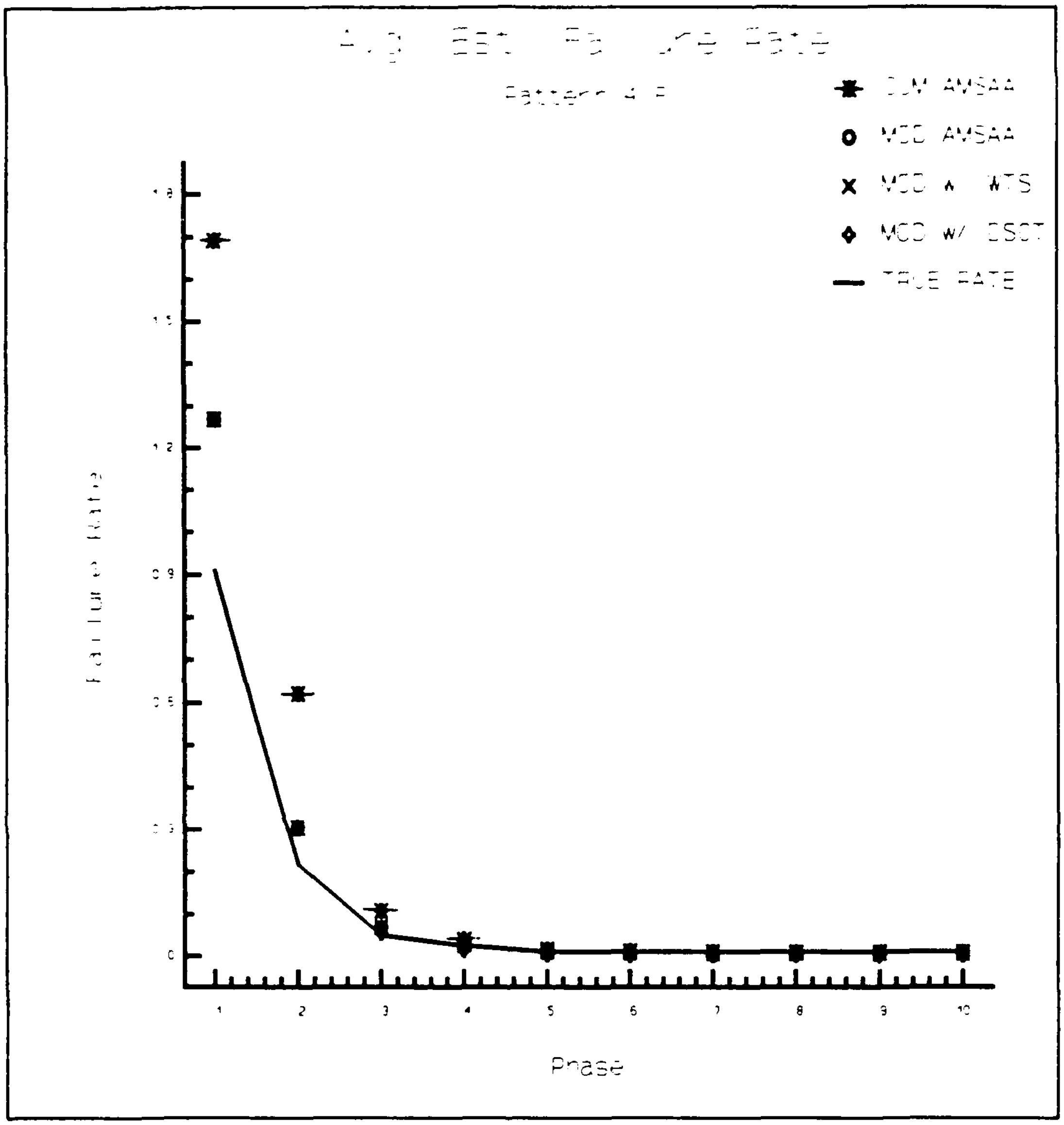

MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
Phase

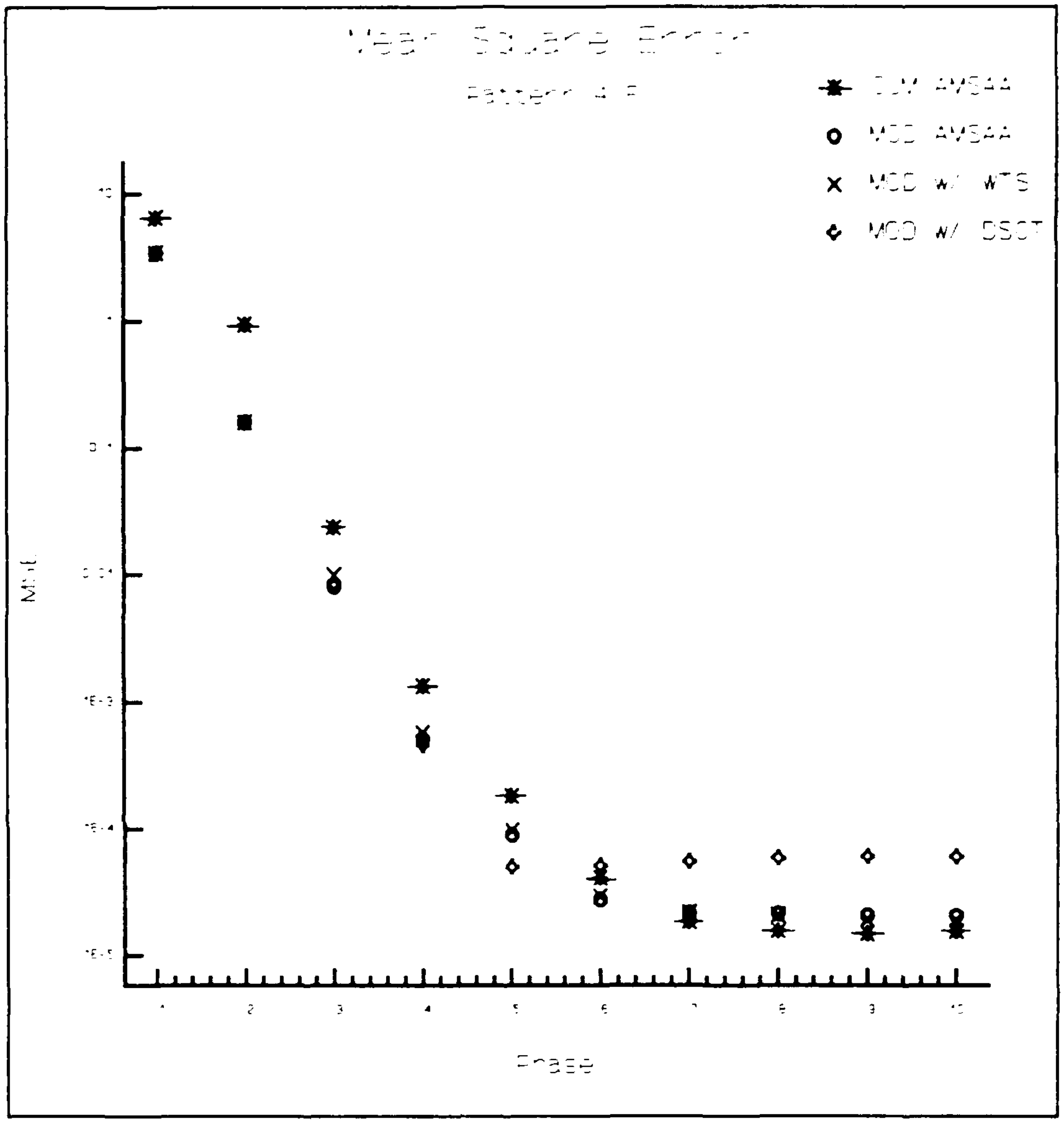

Phase


MOD AMSAA
MOD w/ WTS
MOD w/ DSCT

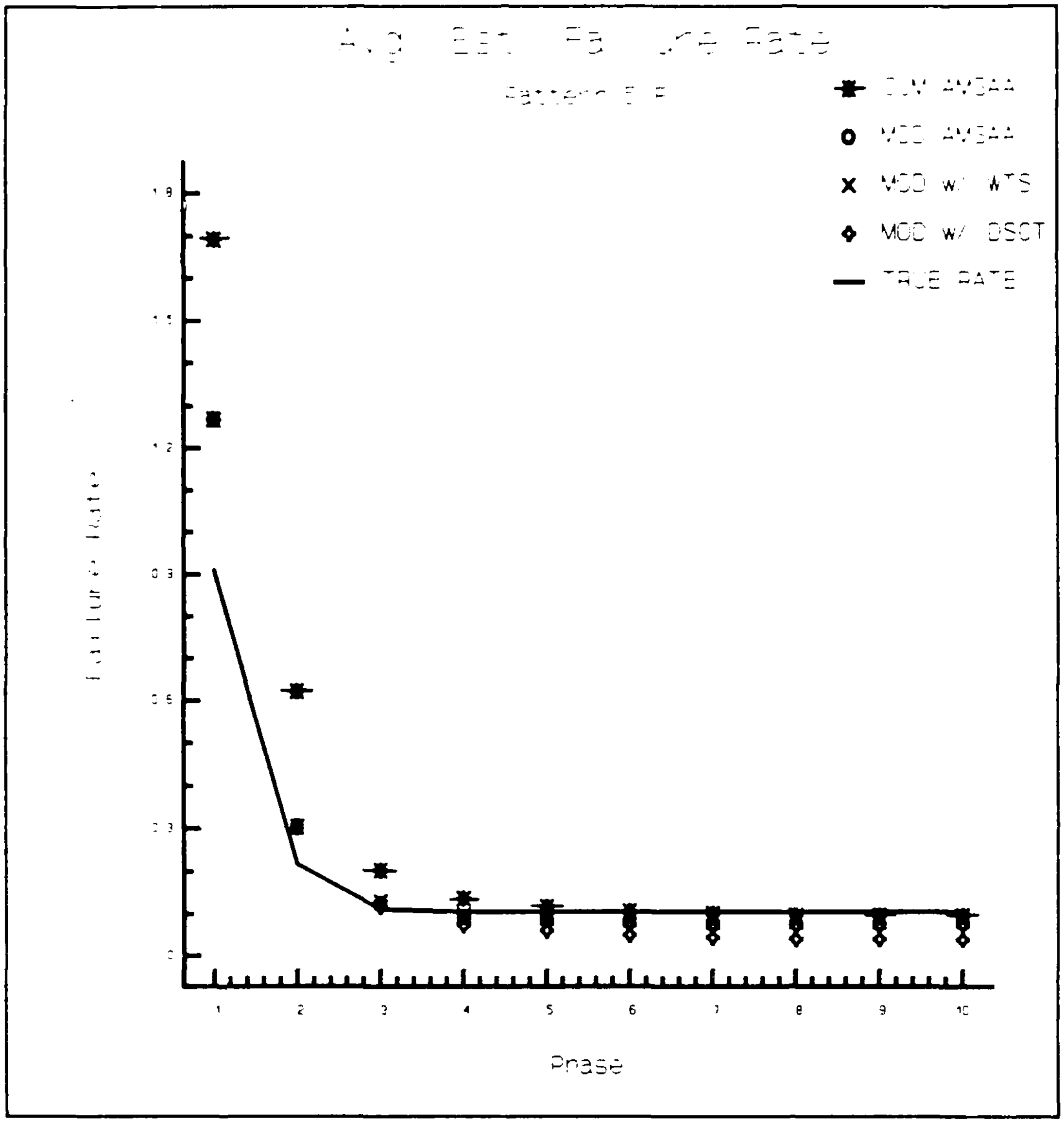


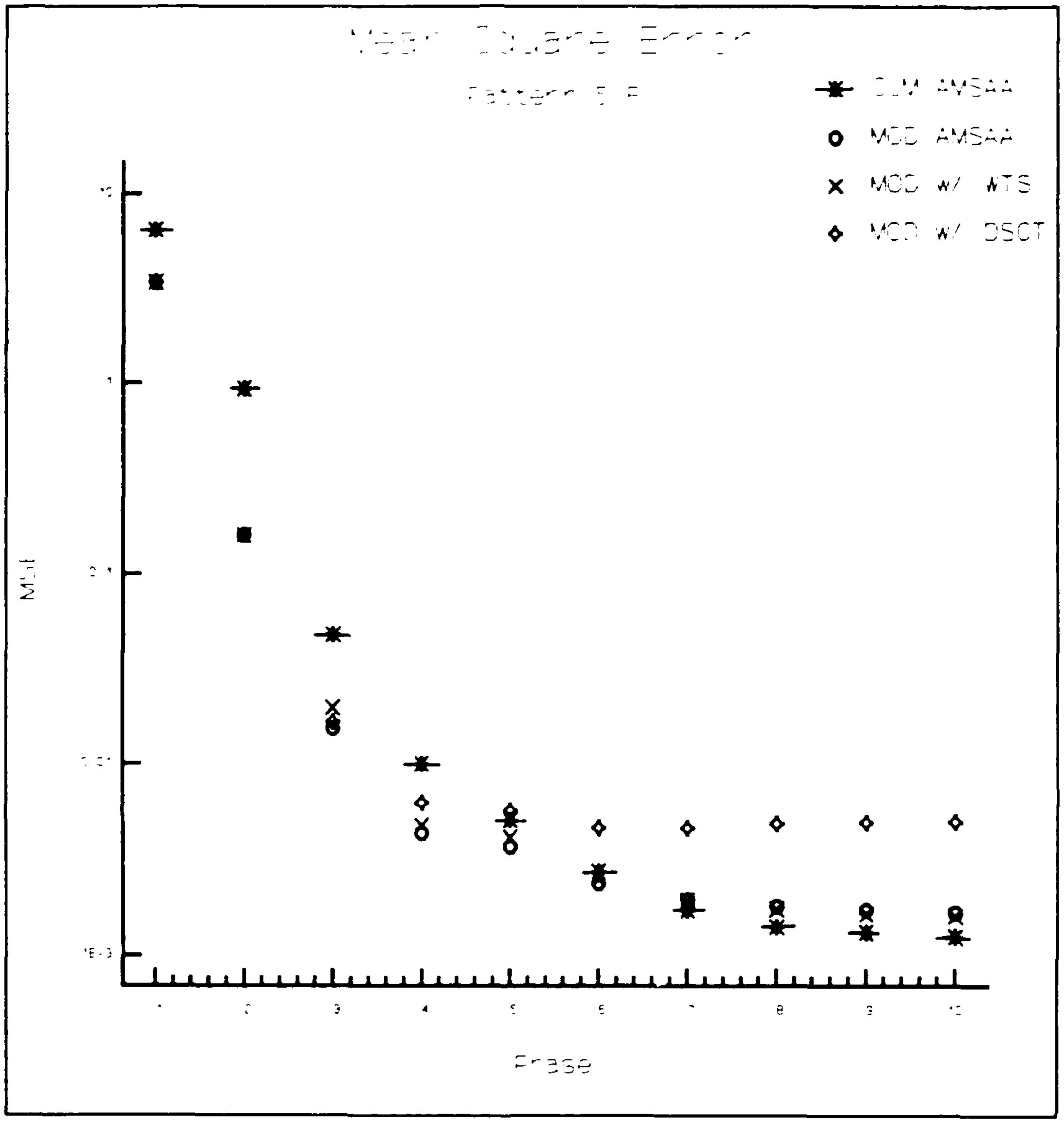

Pattern 3 F
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
10
1
0.1
0.01
1E-3
MSE
1
2
3
4
5
6
7
8
9
10
Phase

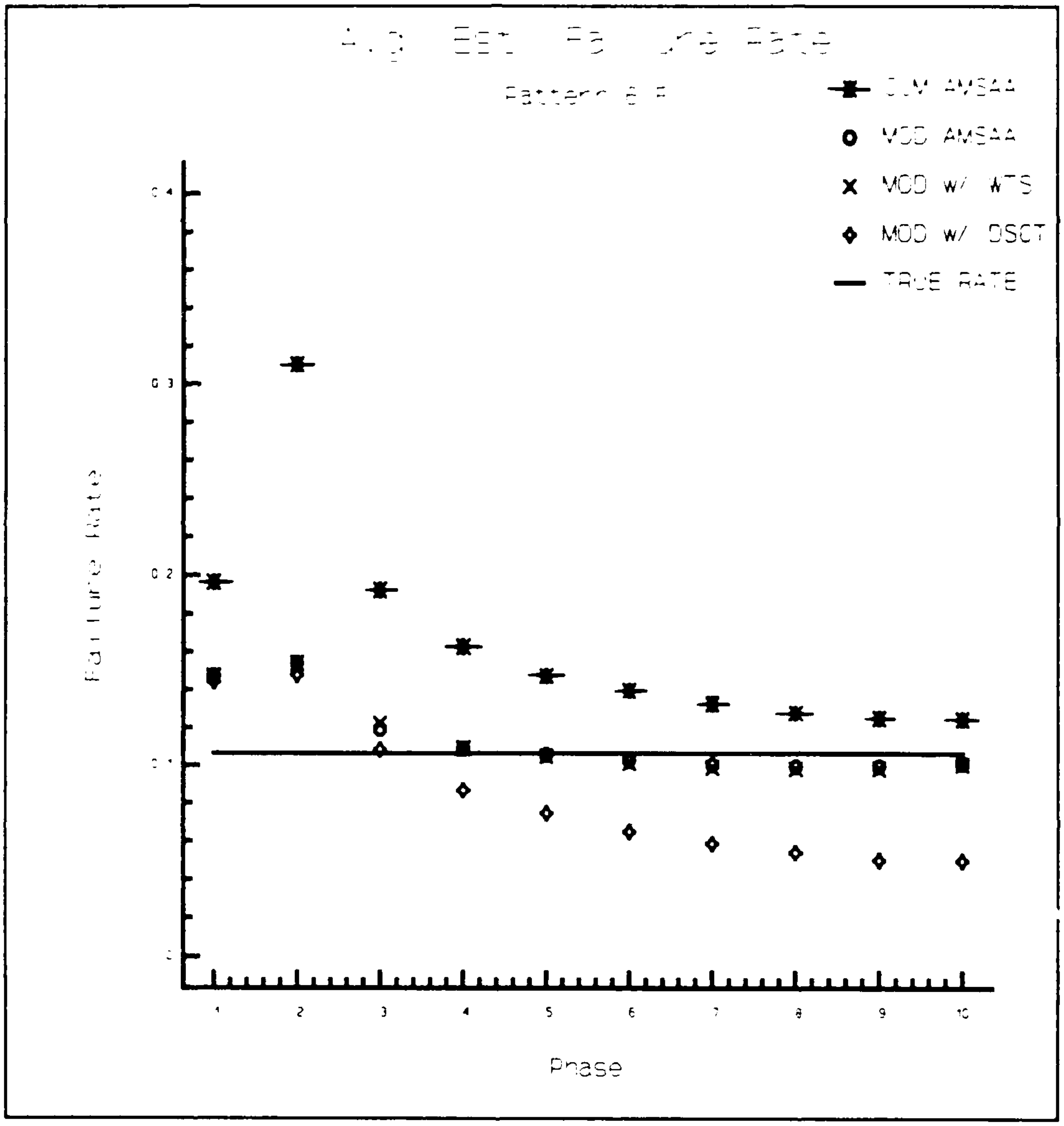

CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase

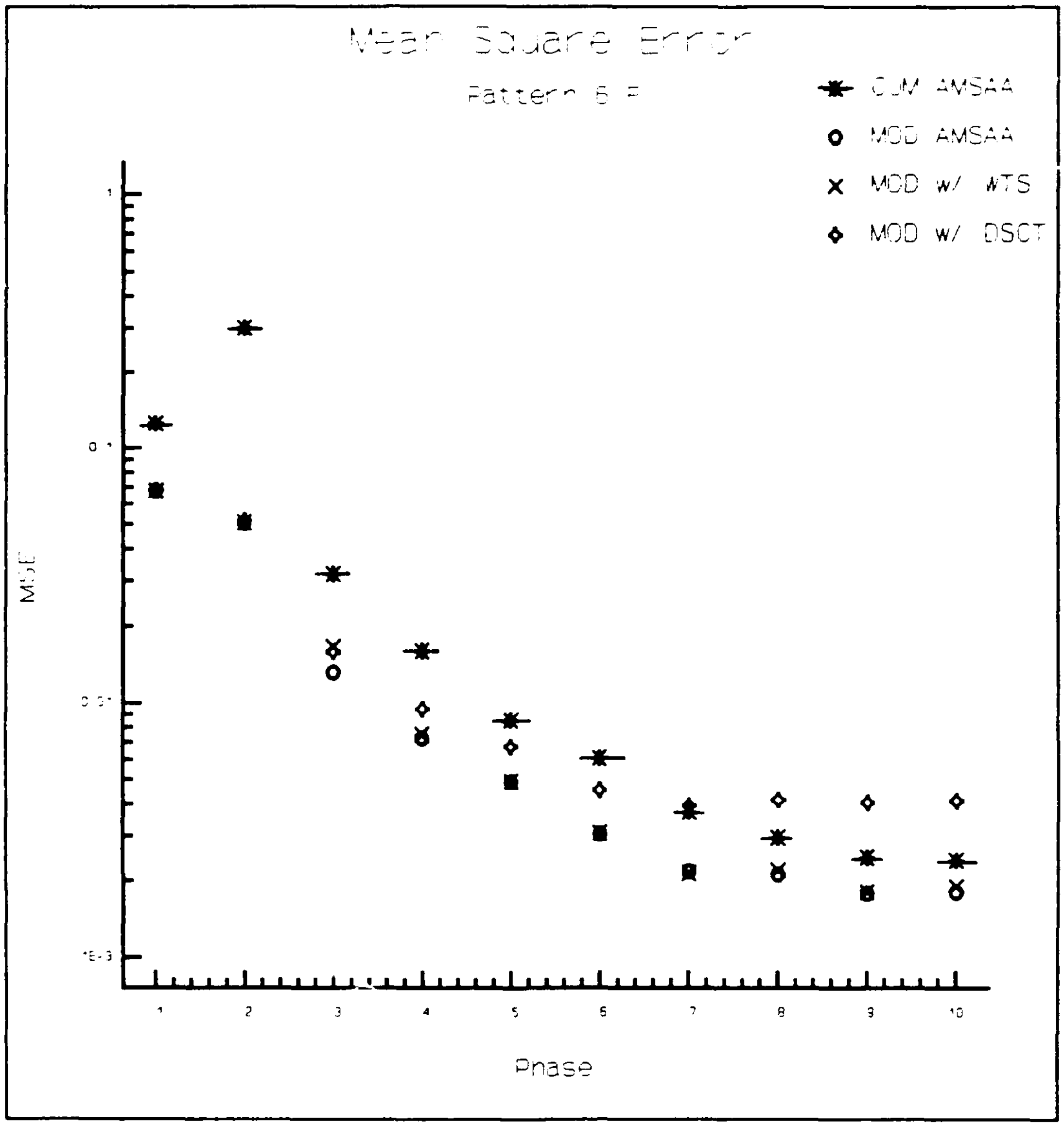


Mean Square Error
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
Phase

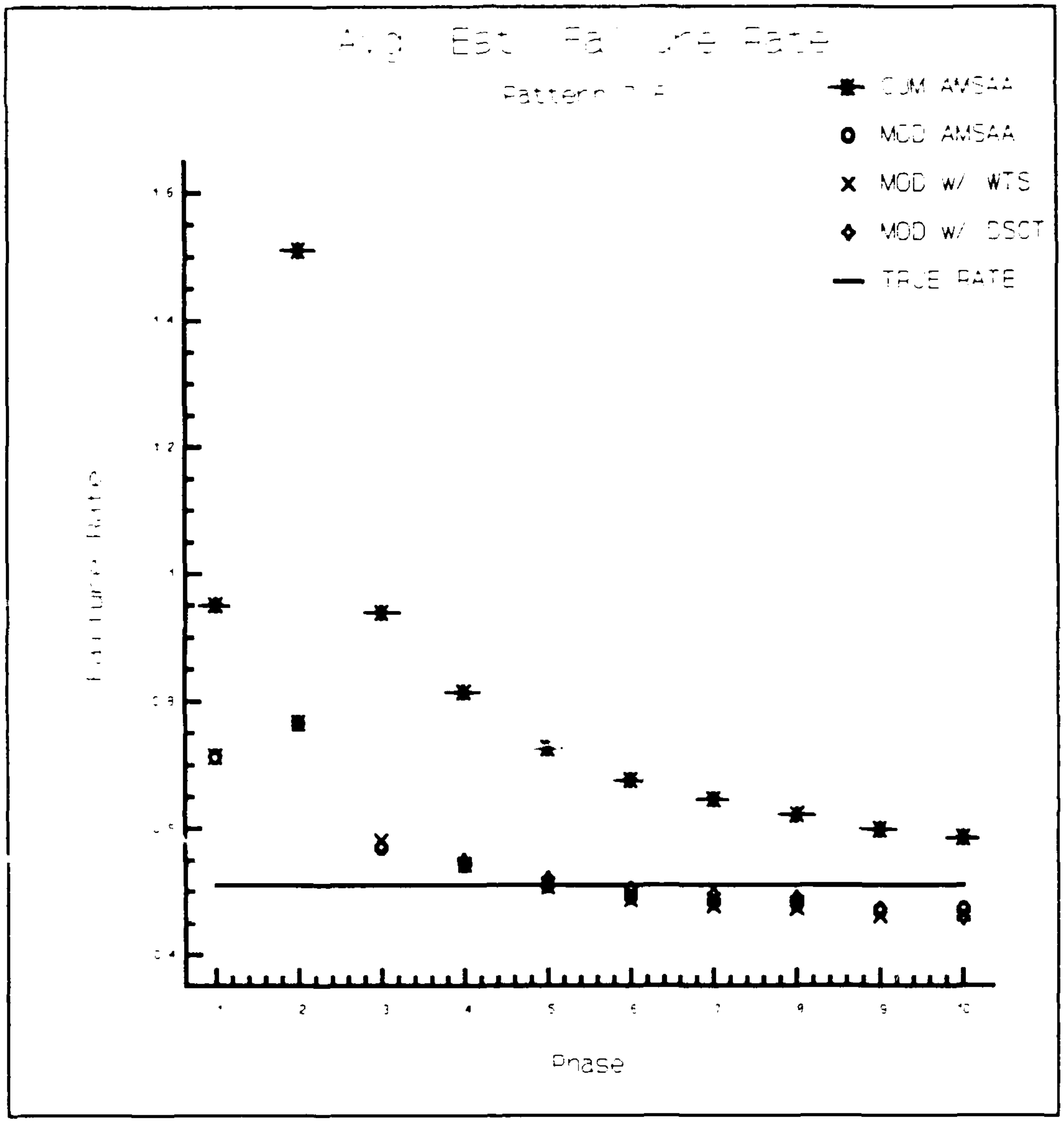

1
2
3
4
5
6
7
8
9
10
Phase

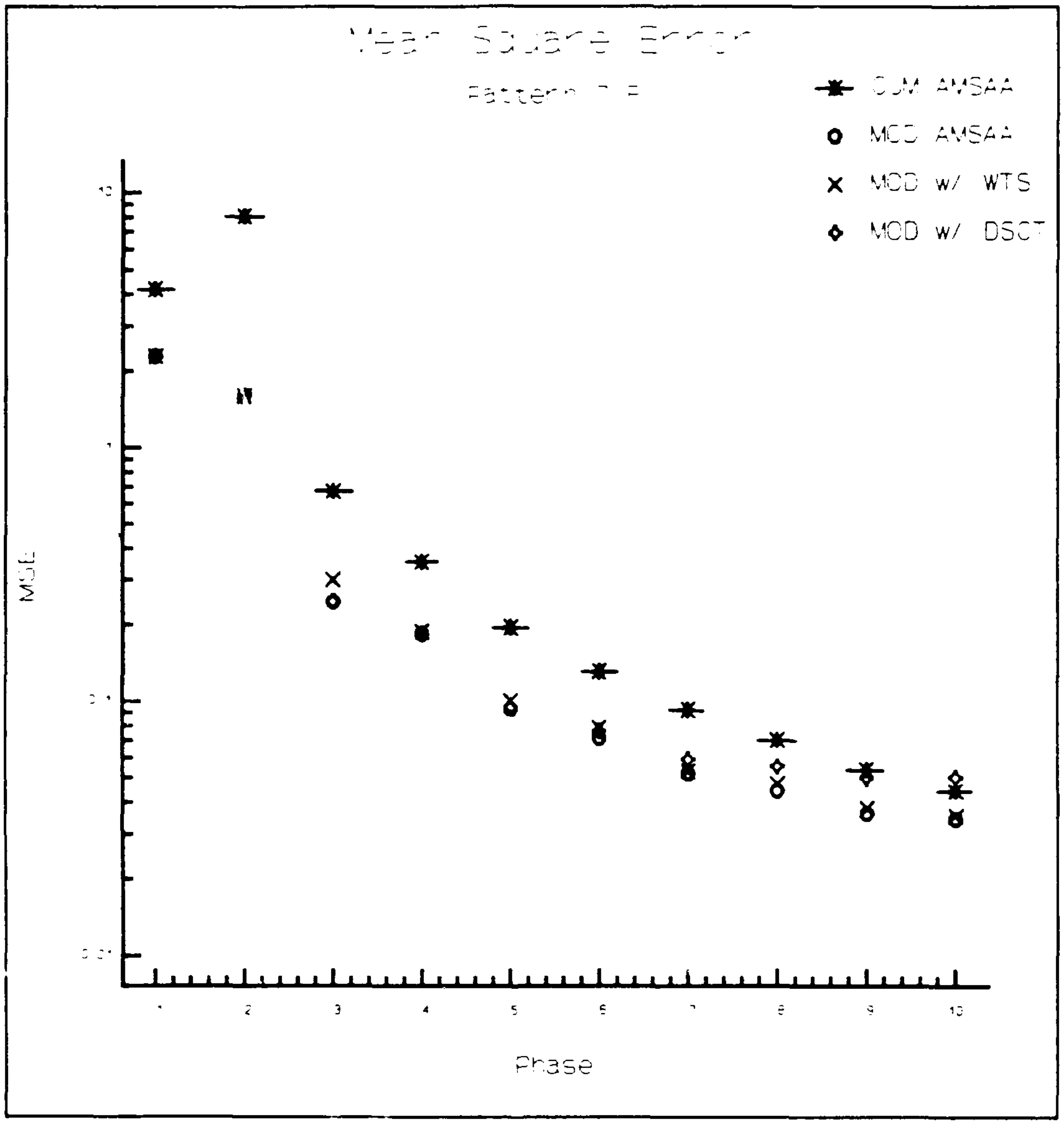

Mean Square Error
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
Phase


MOD w/ DSCT
Failure Rate
0.3
0.2
2
3
4
5
6
8
9
Phase

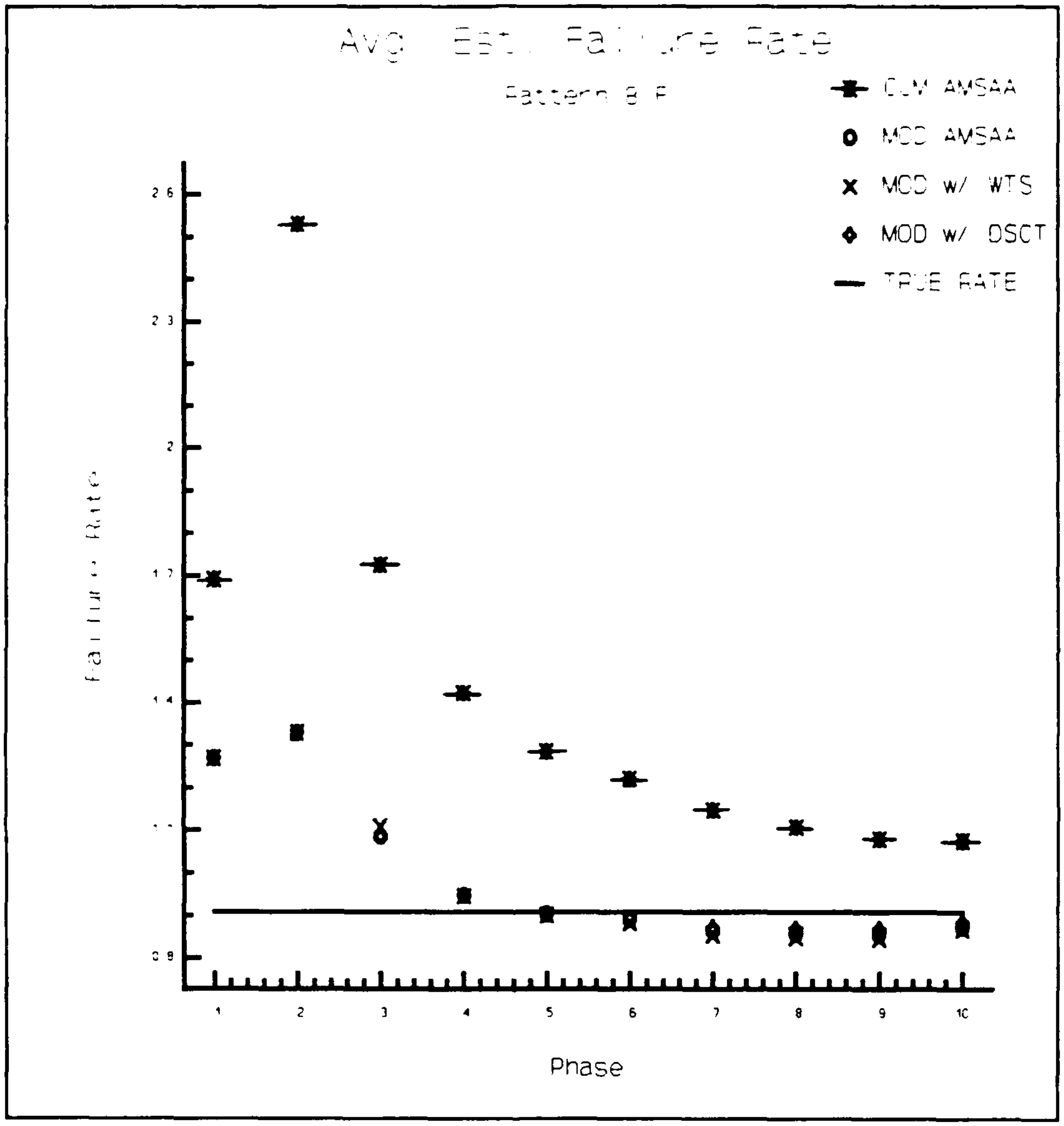


Mean Square Error
Pattern 6 F
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
1
0.1
0.01
1E-3
MSE
1
2
3
4
5
6
7
8
9
10
Phase

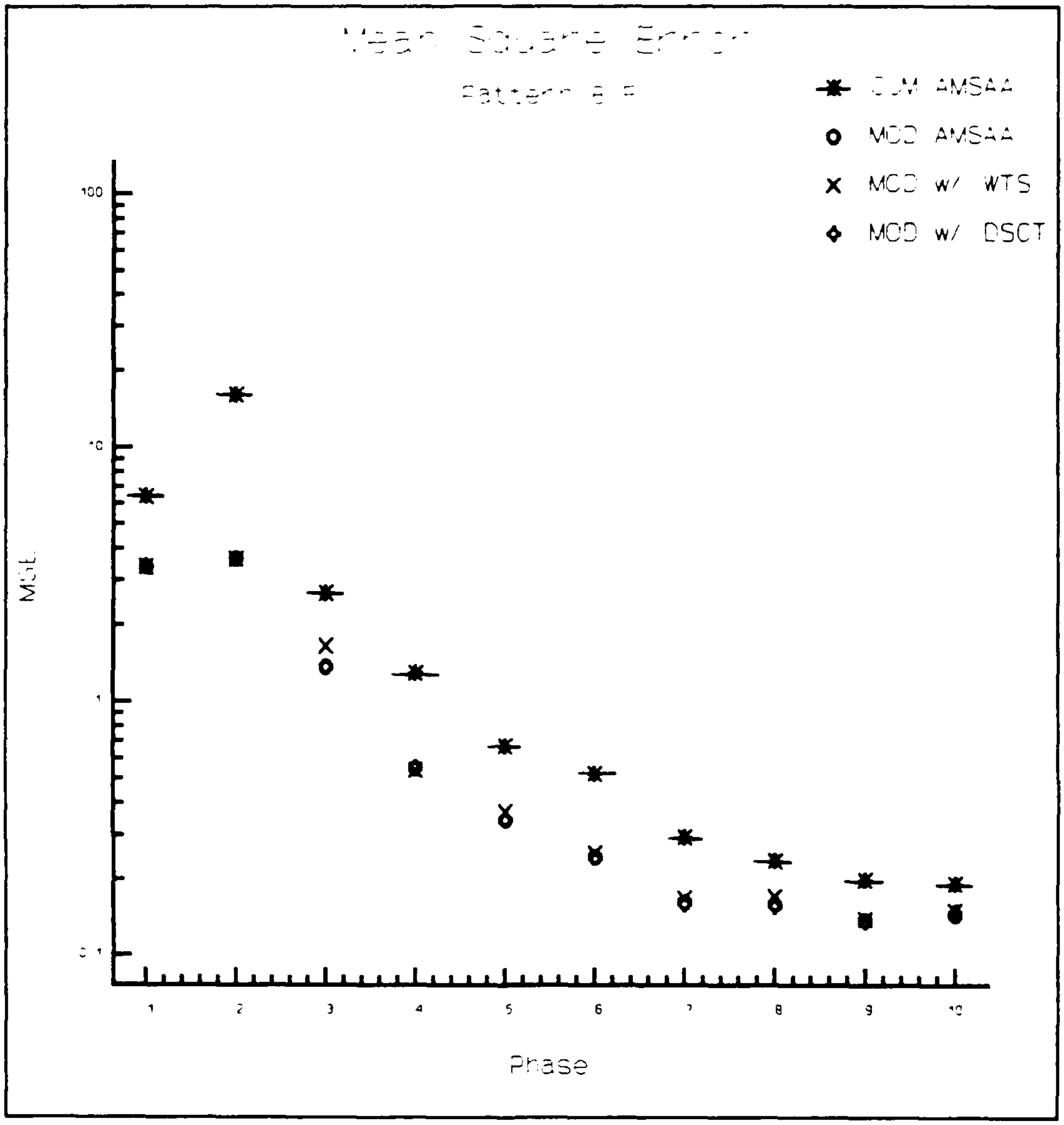

MOD AMSAA
MOD w/ WTS
TRUE RATE
Phase

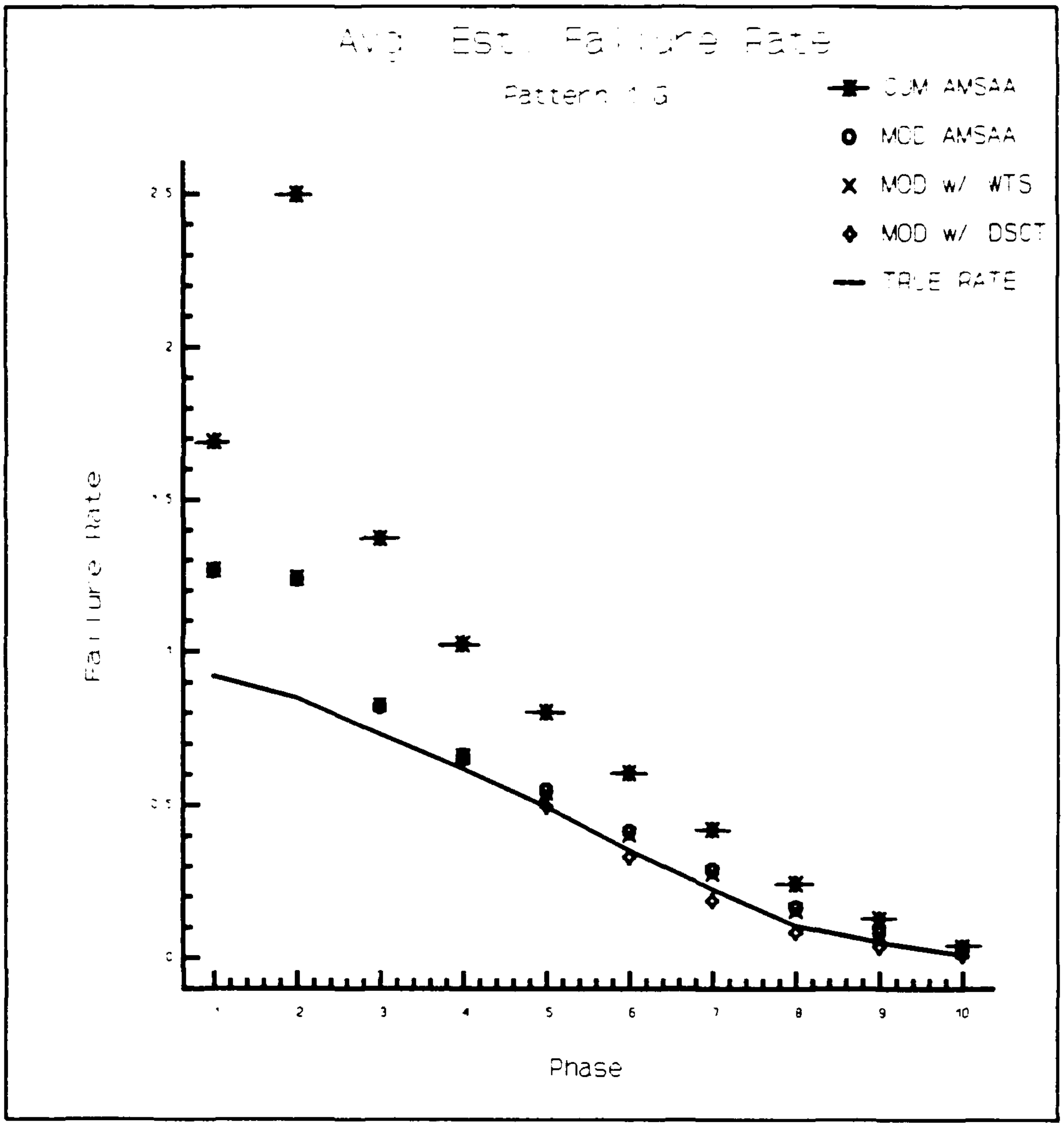


Mean Square Error
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
Phase

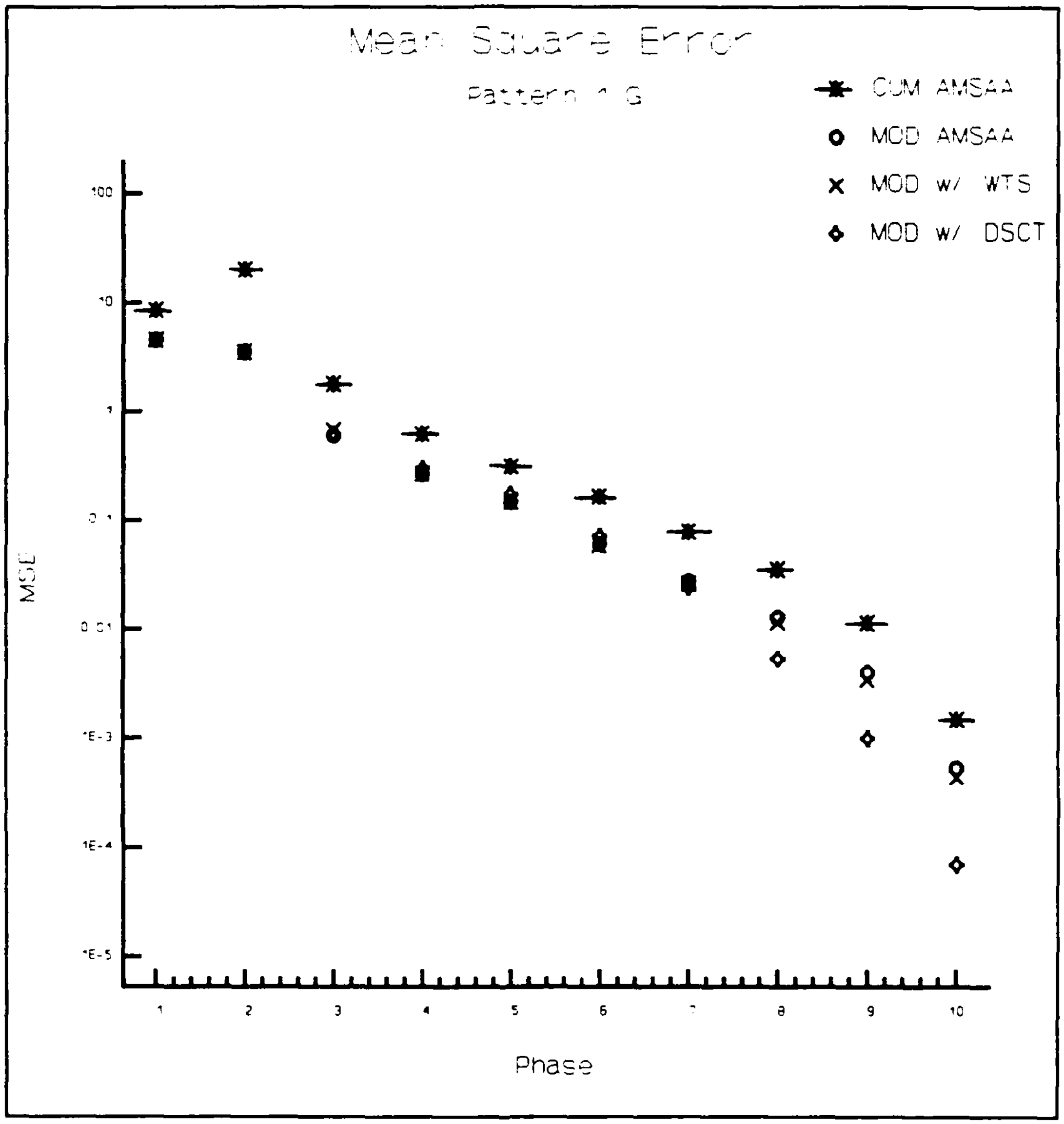


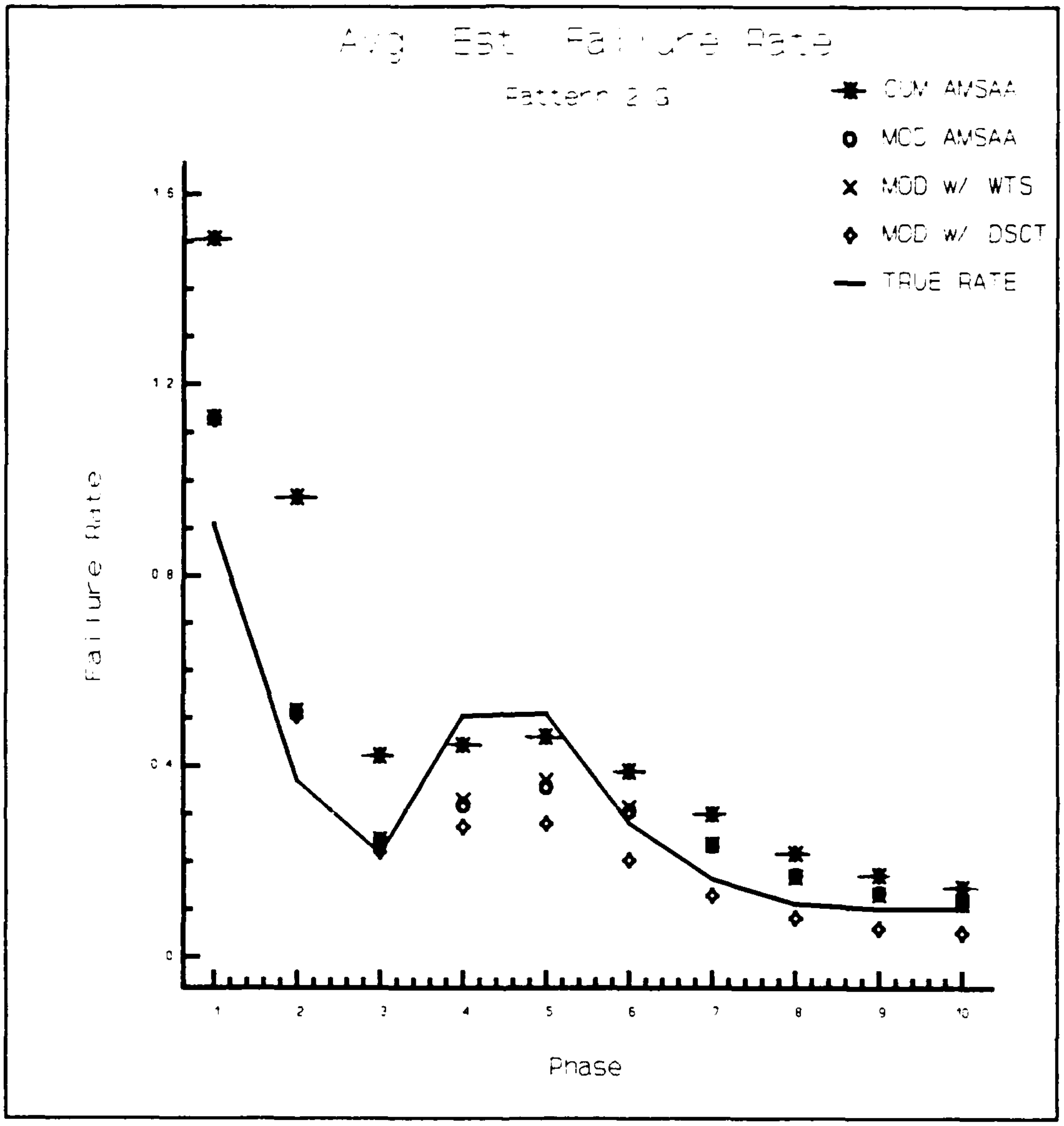
Avg. Est. Failure Rate
Pattern 8 F
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase


Mean Square Error
Pattern 8 F
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
100
10
1
0.1
MSE
1
2
3
4
5
6
7
8
9
10
Phase

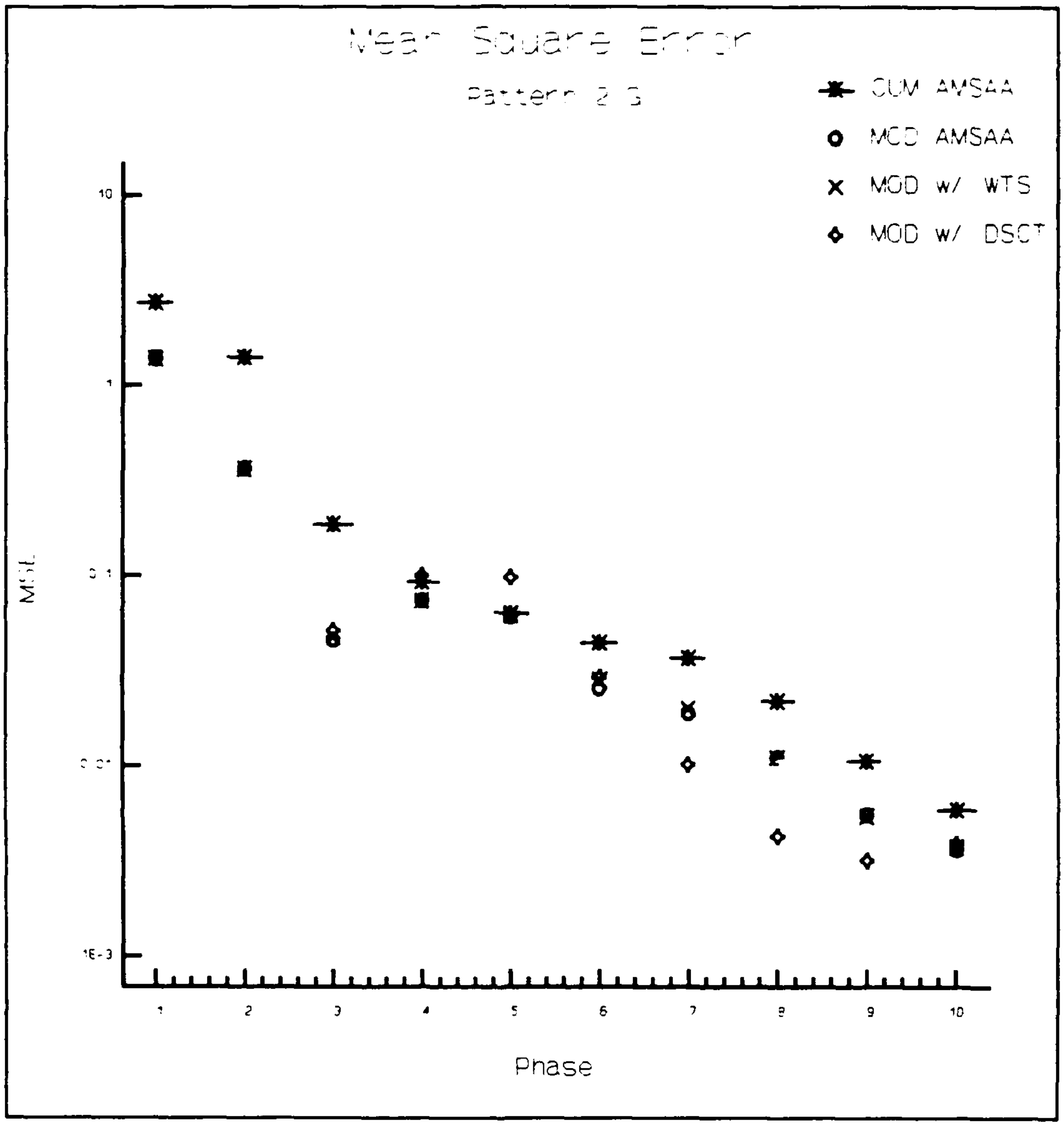


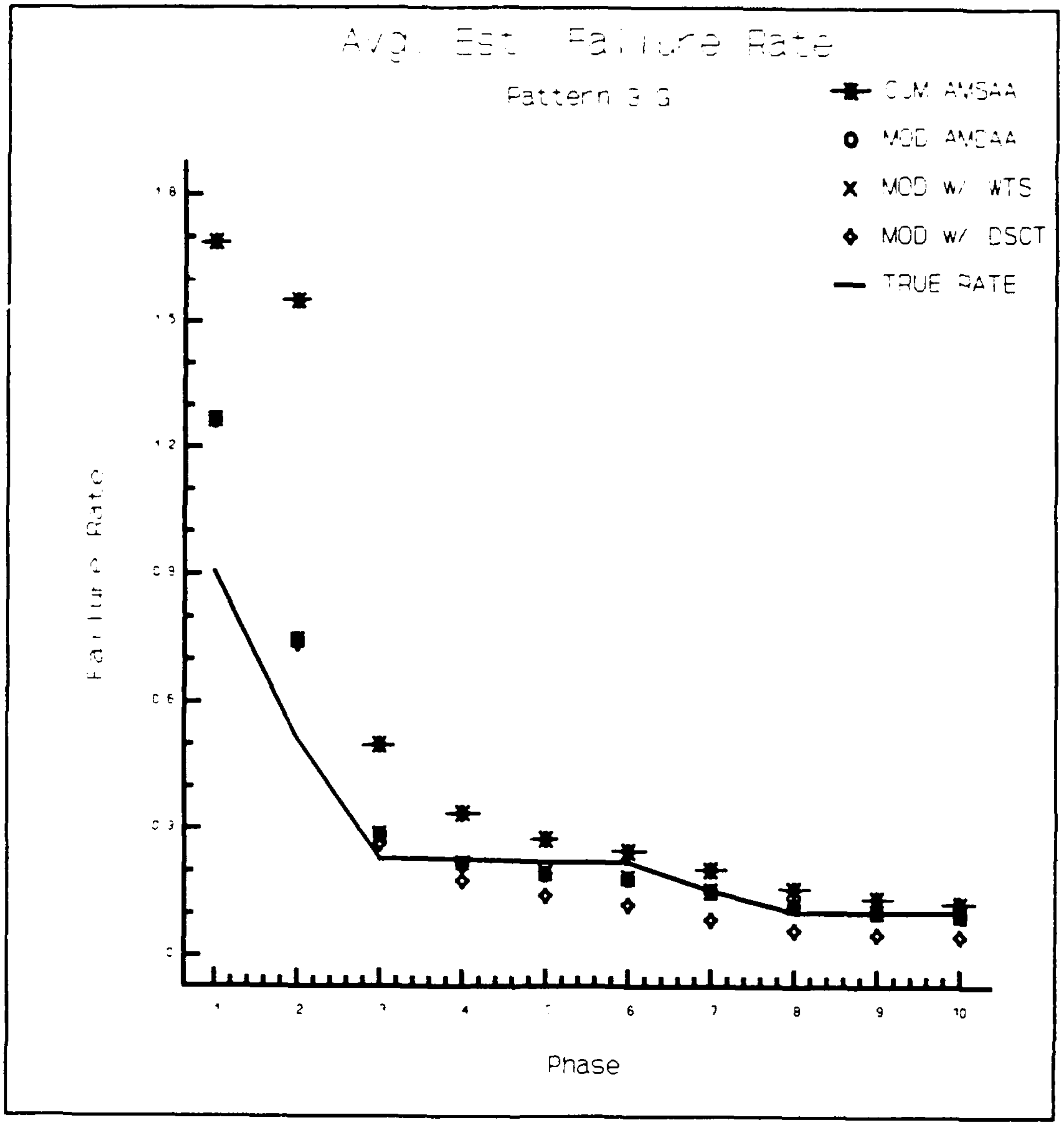
Avg. Est. Failure Rate
Pattern 1.G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase


Mean Square Error
Pattern 1 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
Phase

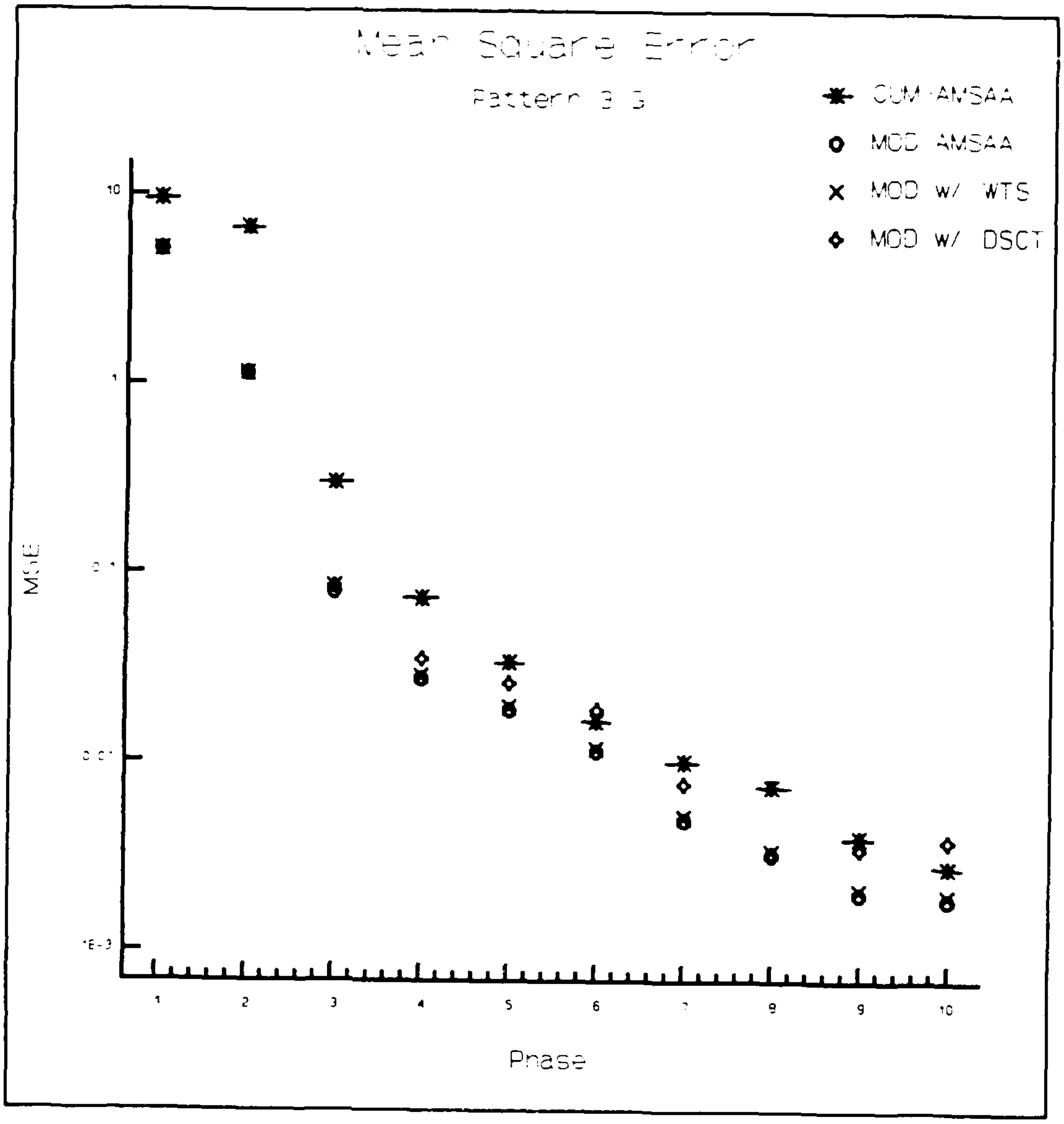


Avg Est Failure Rate
Pattern 2 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
1.6
1.2
0.8
0.4
0
1
2
3
4
5
6
7
8
9
10
Phase

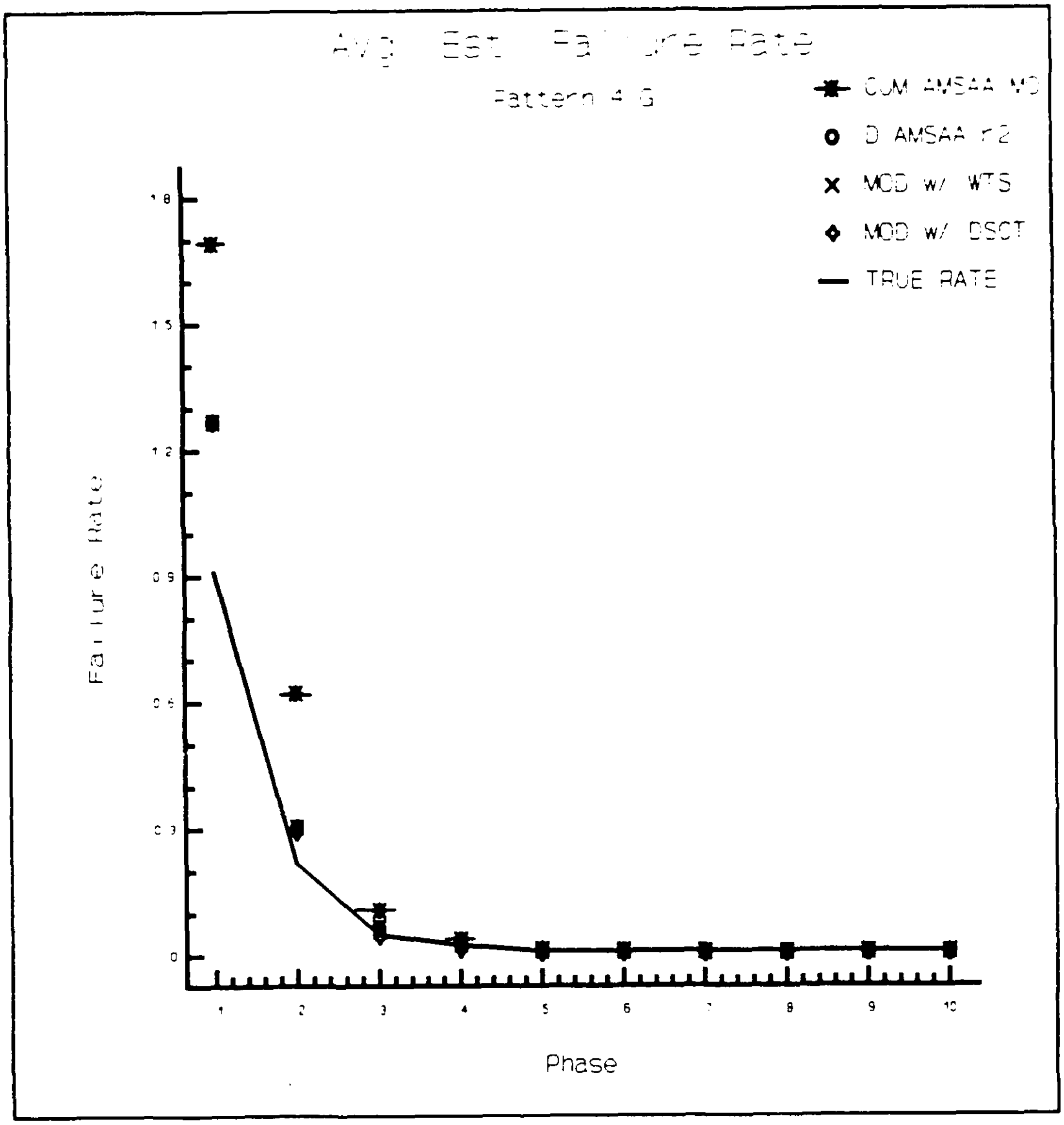

Mean Square Error
Pattern 2 3
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
10
1
0.1
0.01
1E-3
1
2
3
4
5
6
7
8
9
10
Phase

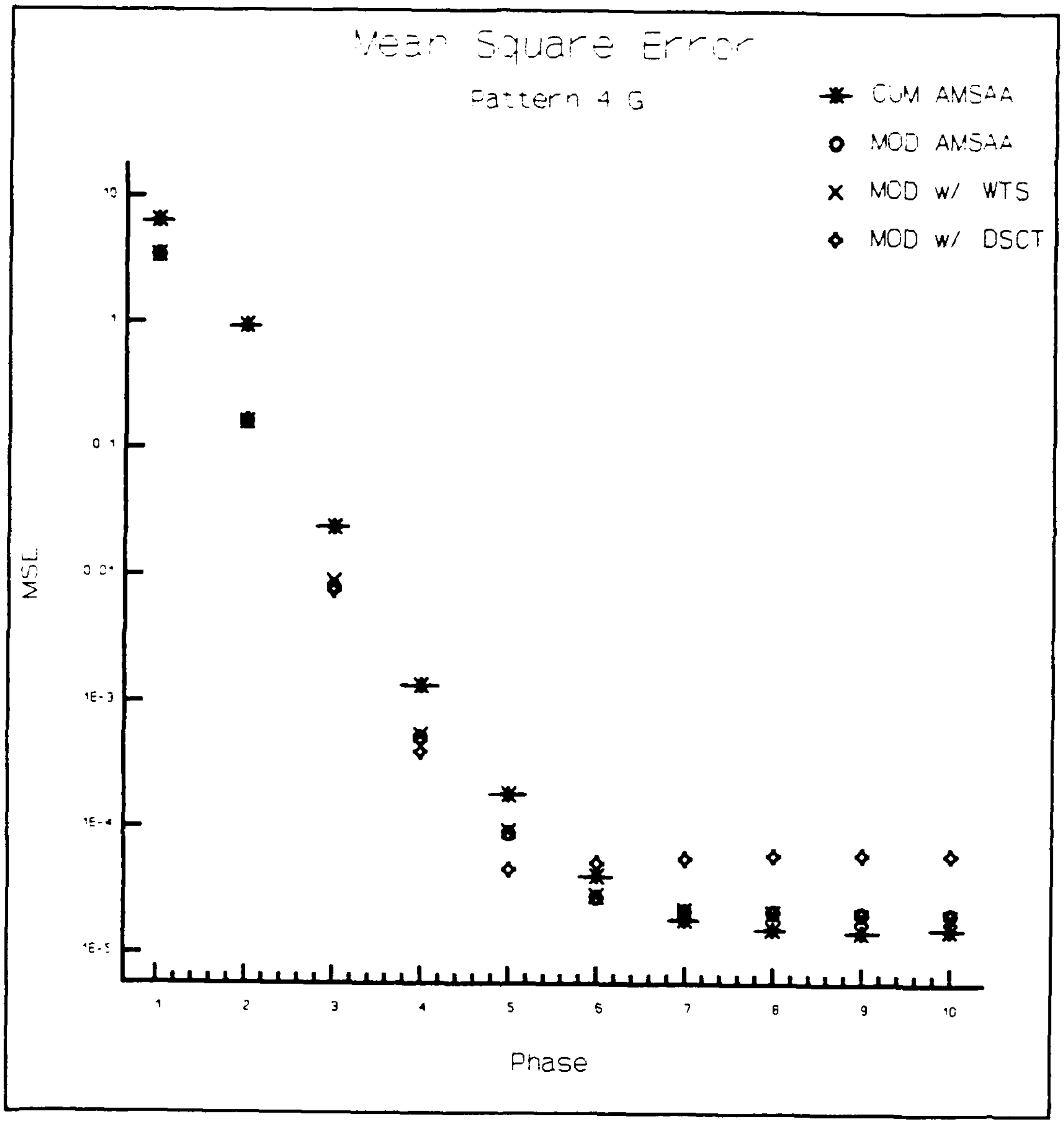


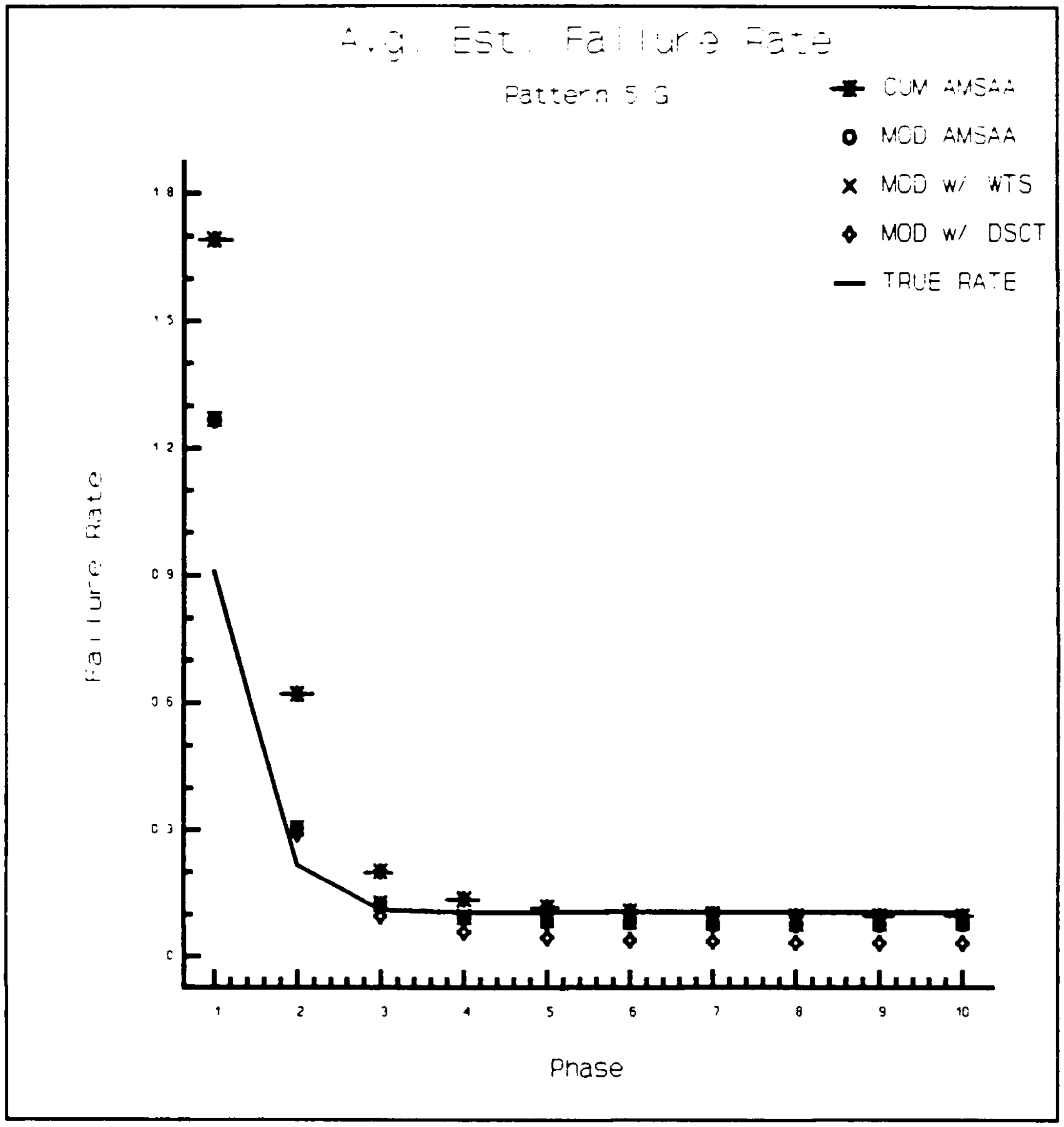
Avg. Est. Failure Rate
Pattern 3 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase



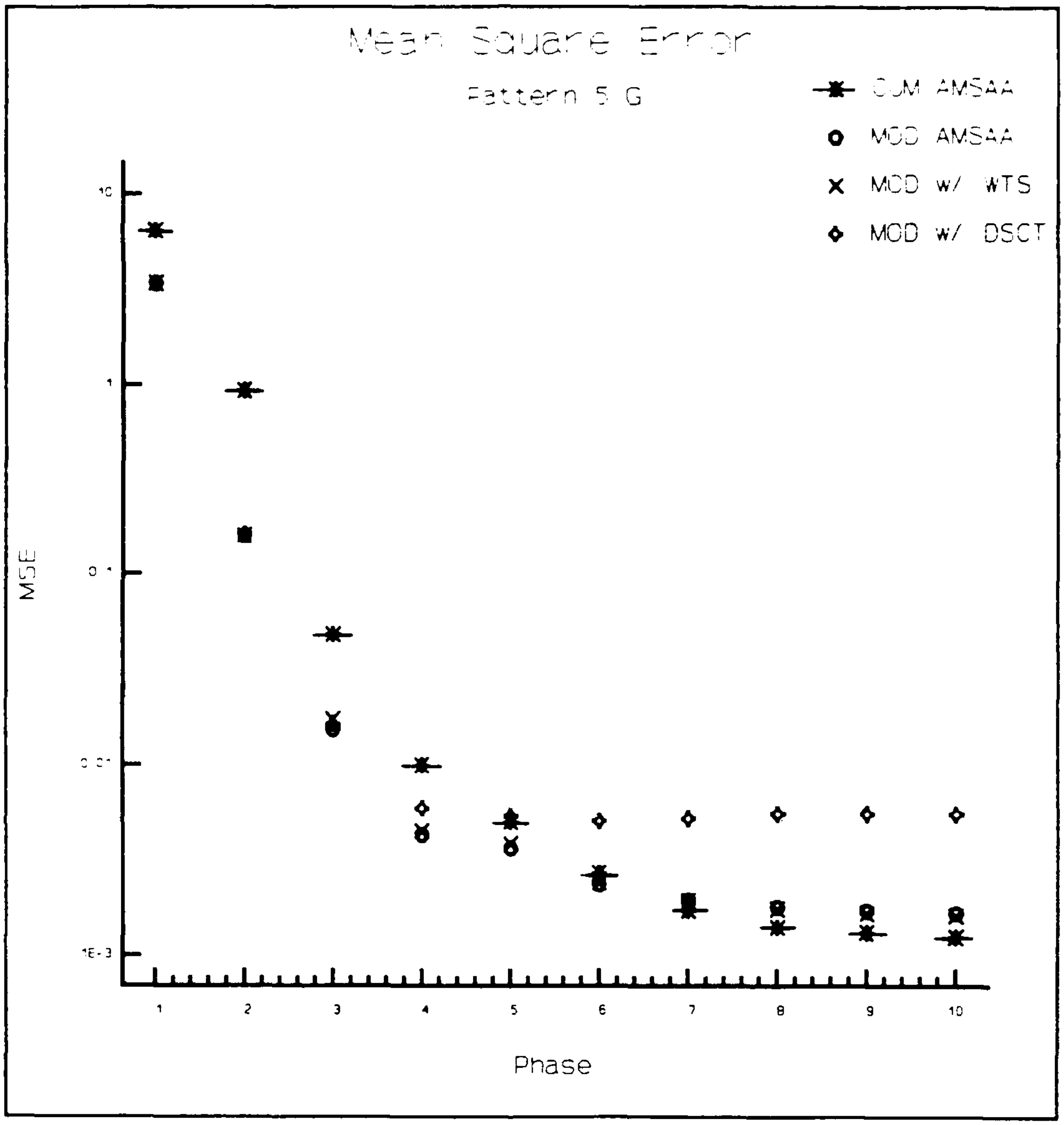
Mean Square Error
Pattern 3 5
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
Phase


Avg. Est. Failure Rate
Pattern 4 G
CUM AMSAA MO
D AMSAA r2
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase

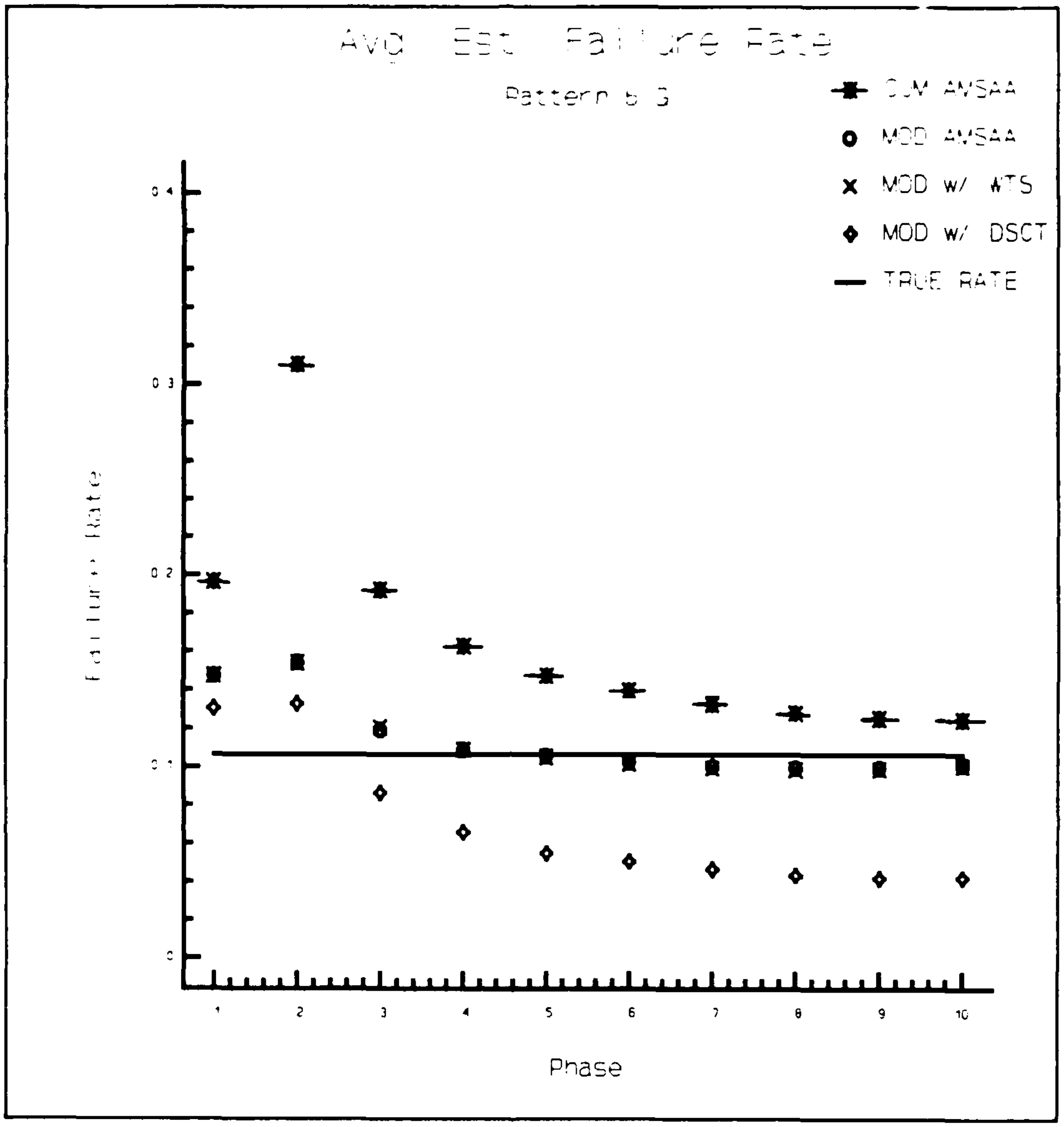

Mean Square Error
Pattern 4 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
10
1
0.1
0.01
1E-3
1E-4
1E-5
1
2
3
4
5
6
7
8
9
10
Phase

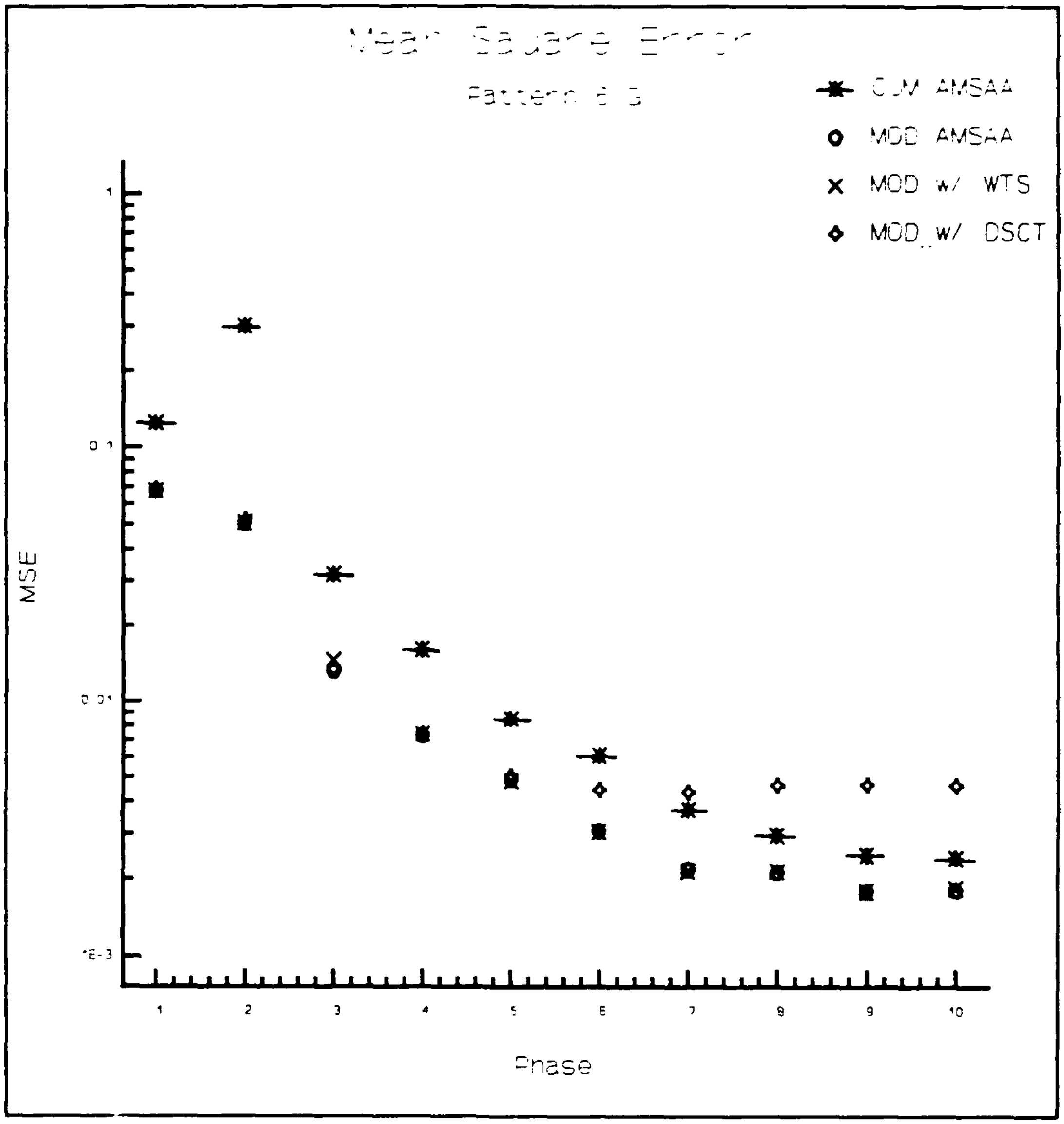


Avg. Est. Failure Rate
Pattern 5 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
1.8
1.5
1.2
0.9
0.6
0.3
0
1
2
3
4
5
6
7
8
9
10
Phase

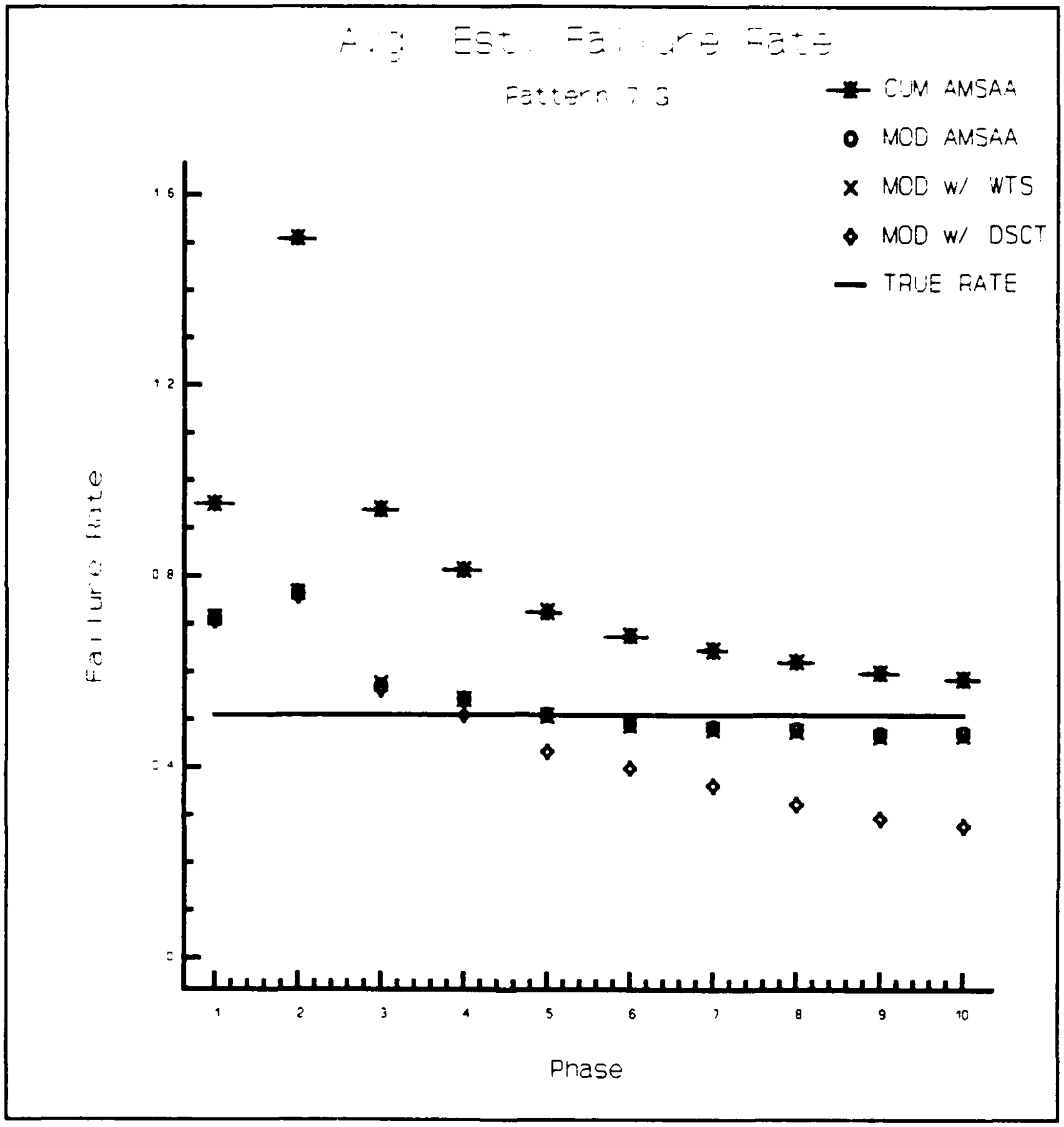


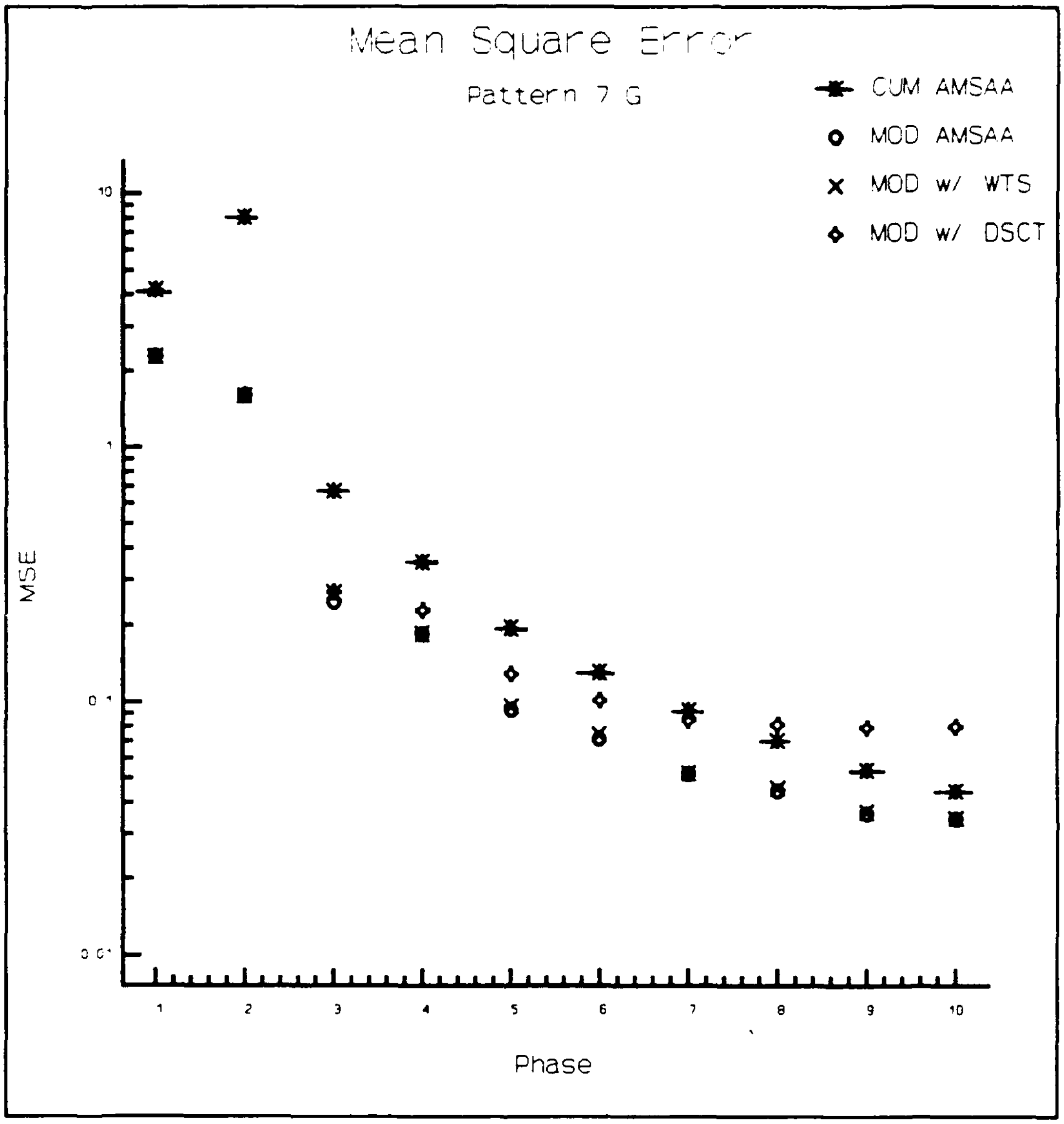
Mean Square Error
Pattern 5 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
10
1
0.1
0.01
1E-3
1
2
3
4
5
6
7
8
9
10
Phase


Avg Est Failure Rate
Pattern 5 3
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
0.4
0.3
0.2
0.1
0
Failure Rate
1
2
3
4
5
6
7
8
9
10
Phase

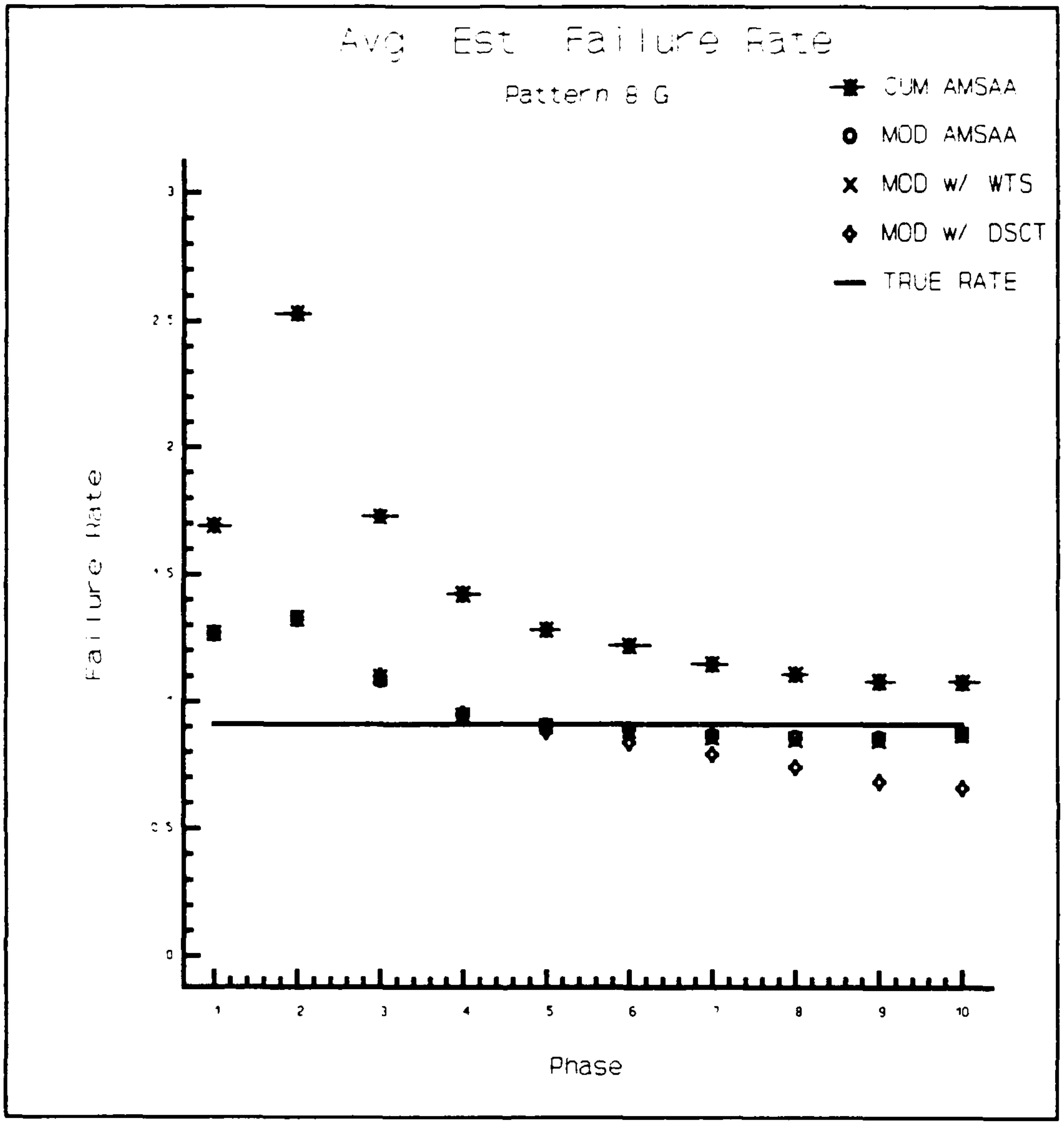


Mean Square Error
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
1
0.1
0.01
1E-3
1
2
3
4
5
6
7
8
9
10
Phase

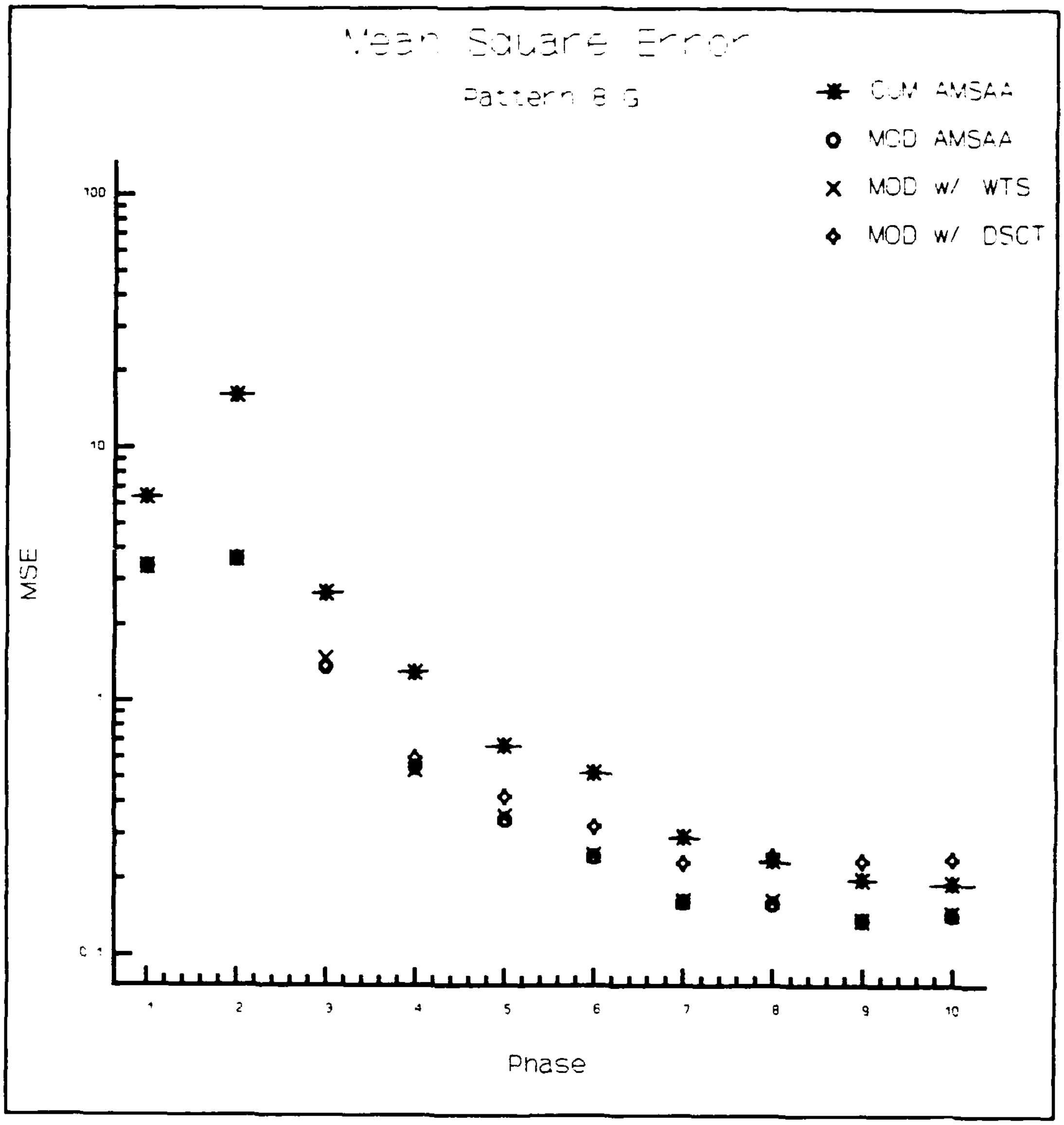

Avg. Est. Failure Rate
Pattern 7 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
1.6
1.2
0.8
0.4
0
Failure Rate
1
2
3
4
5
6
7
8
9
10
Phase

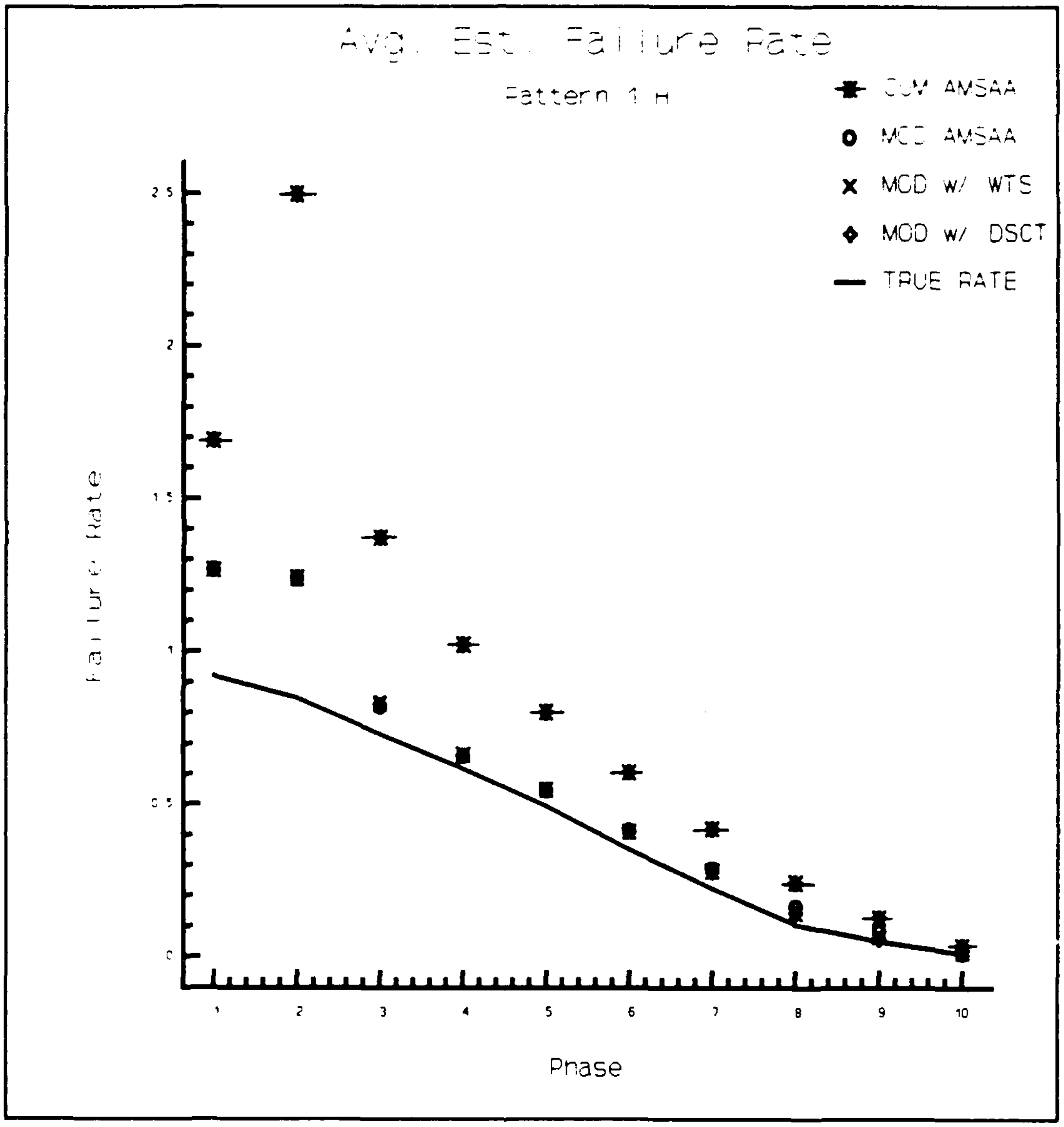


Mean Square Error
Pattern 7 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
10
1
0.1
0.01
1
2
3
4
5
6
7
8
9
10
Phase

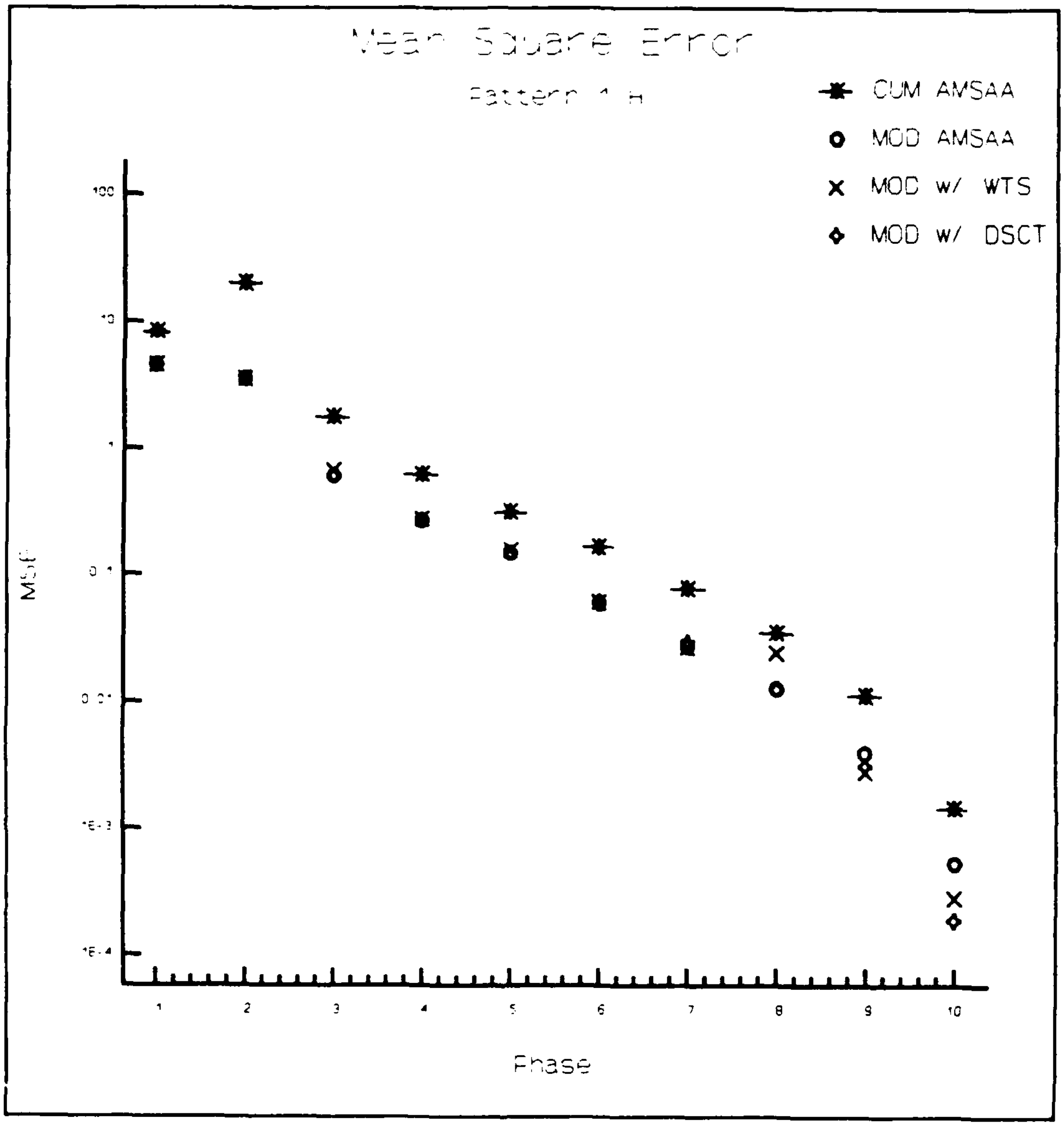


Avg Est Failure Rate
Pattern 8 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase

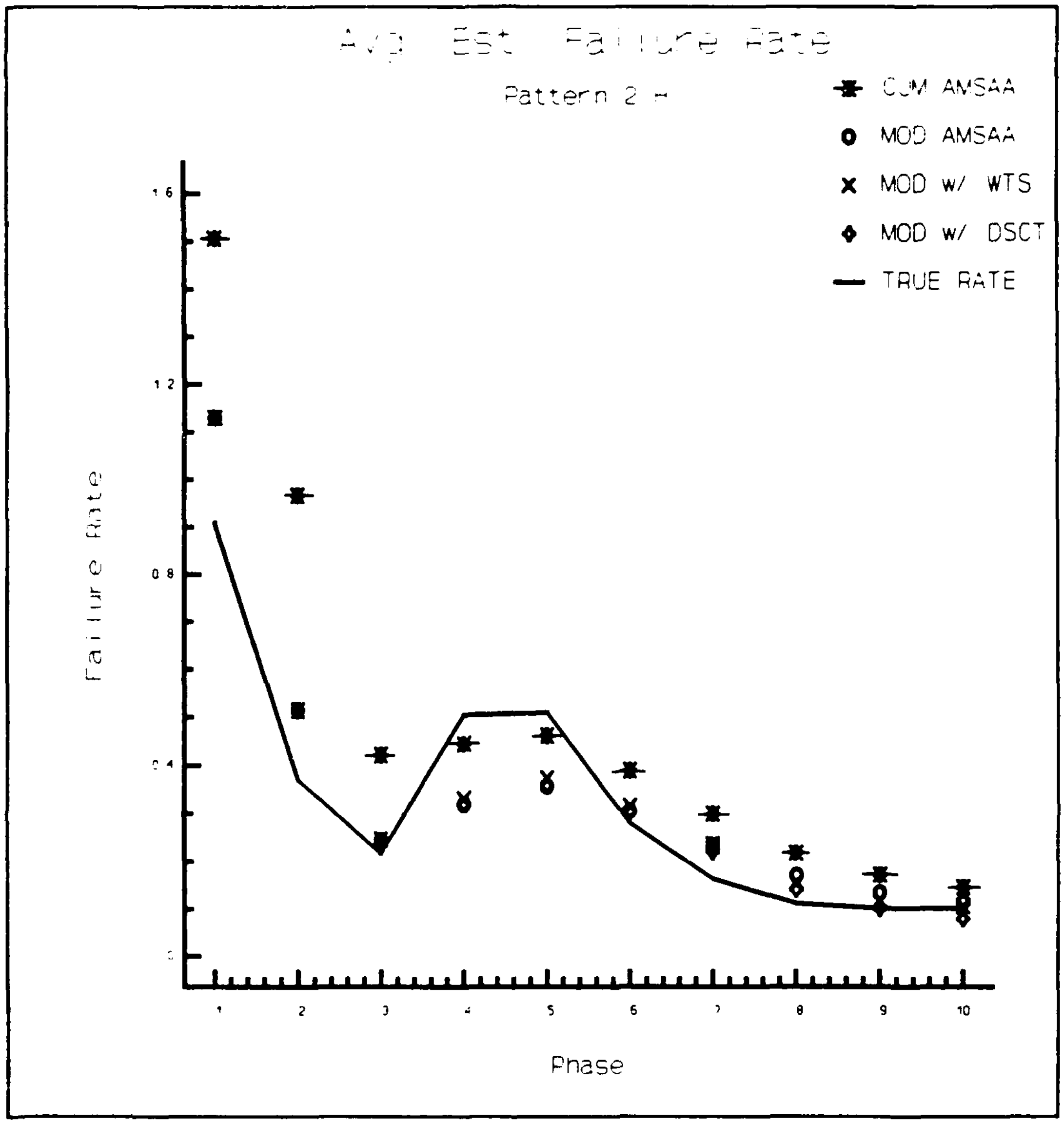

Mean Square Error
Pattern 8 G
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
100
10
1
0.1
MSE
1
2
3
4
5
6
7
8
9
10
Phase



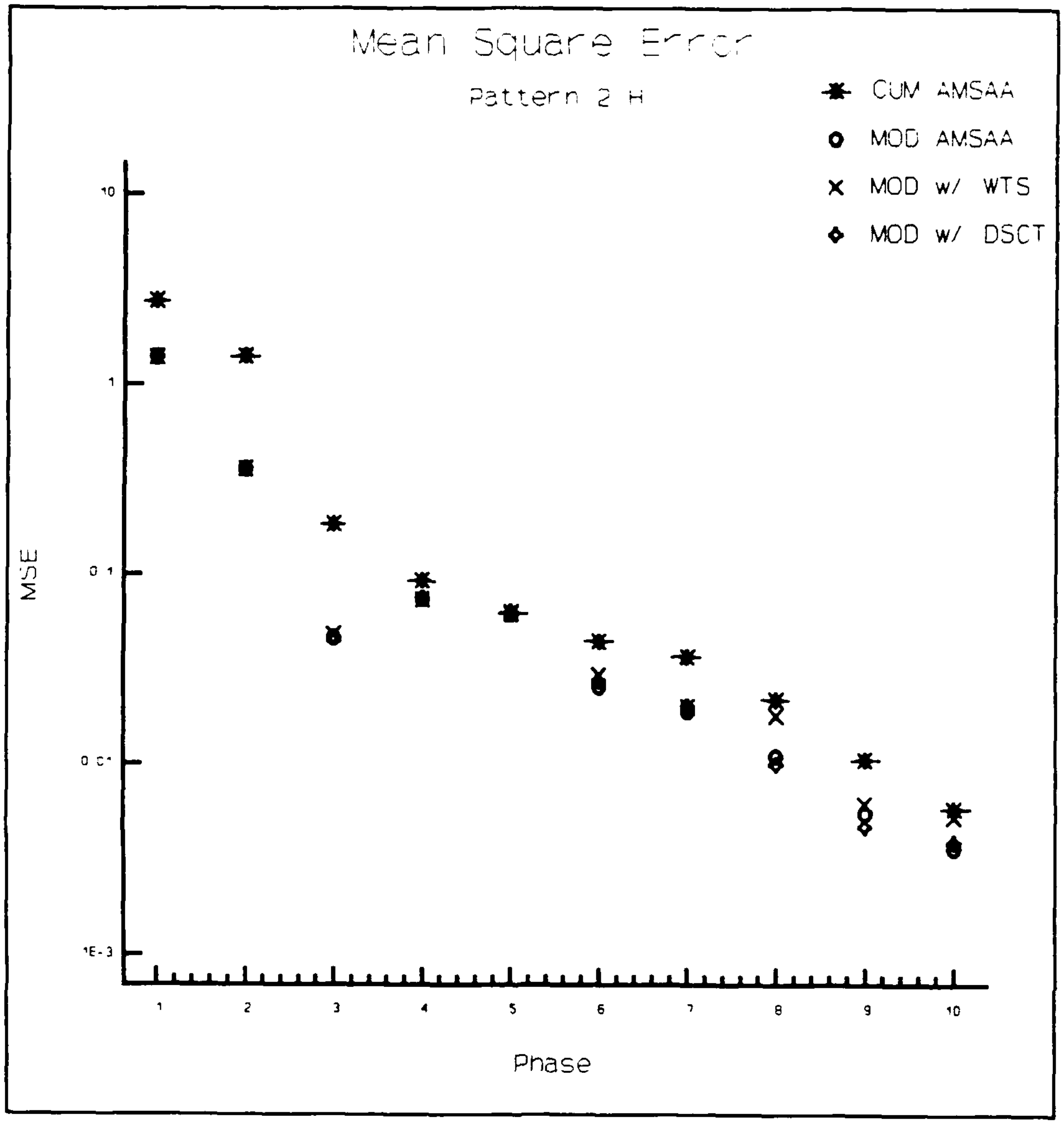
Avg. Est. Failure Rate
Pattern 1 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
2.5
2
1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
10
Phase



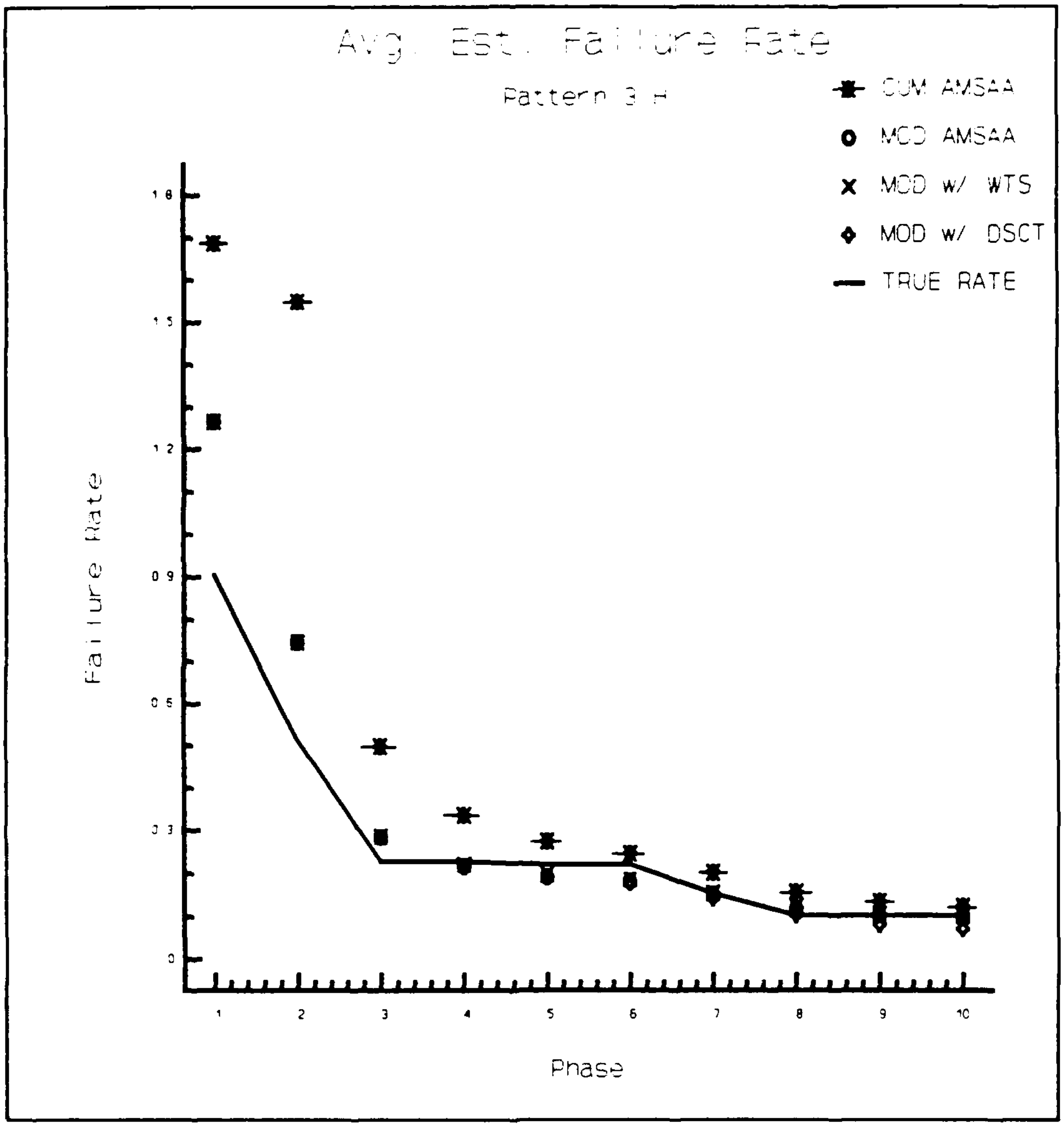
Mean Square Error
Pattern 1 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
Phase
1
2
3
4
5
6
7
8
9
10



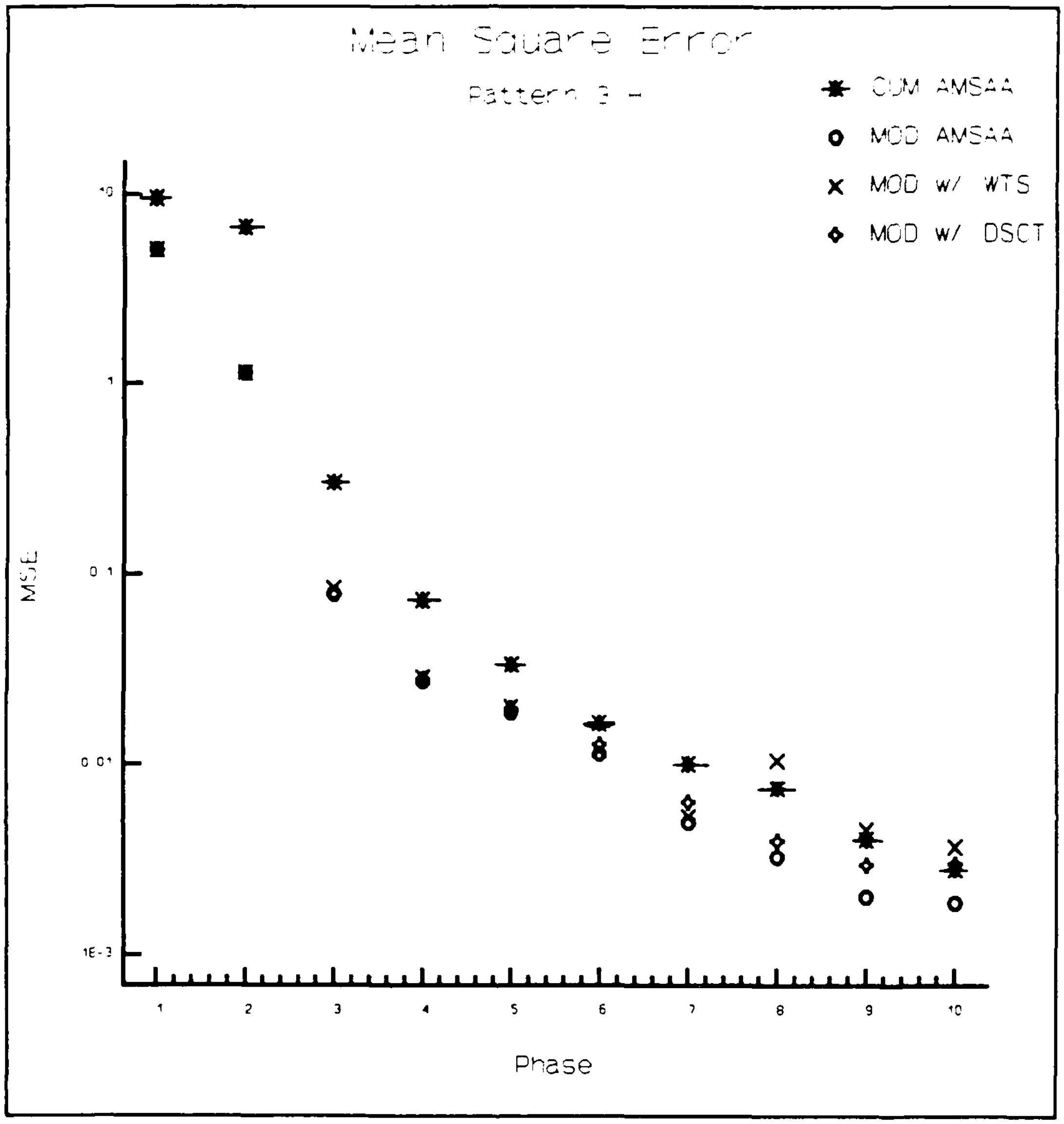
Avg Est Failure Rate
Pattern 2 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
1.6
1.2
0.8
0.4
0
1
2
3
4
5
6
7
8
9
10
Phase



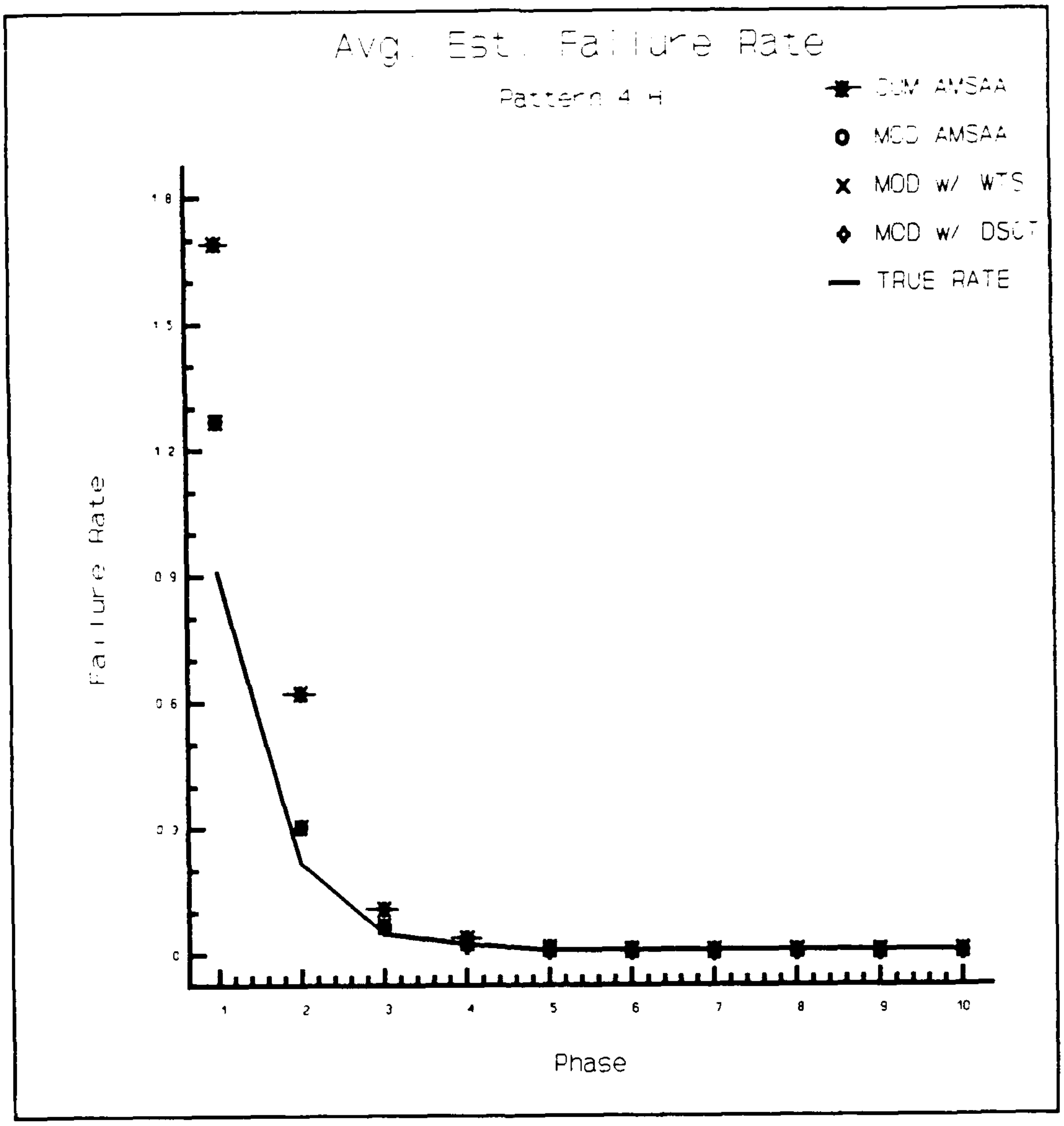
Mean Square Error
Pattern 2 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
10
1
0.1
0.01
1E-3
1
2
3
4
5
6
7
8
9
10
Phase



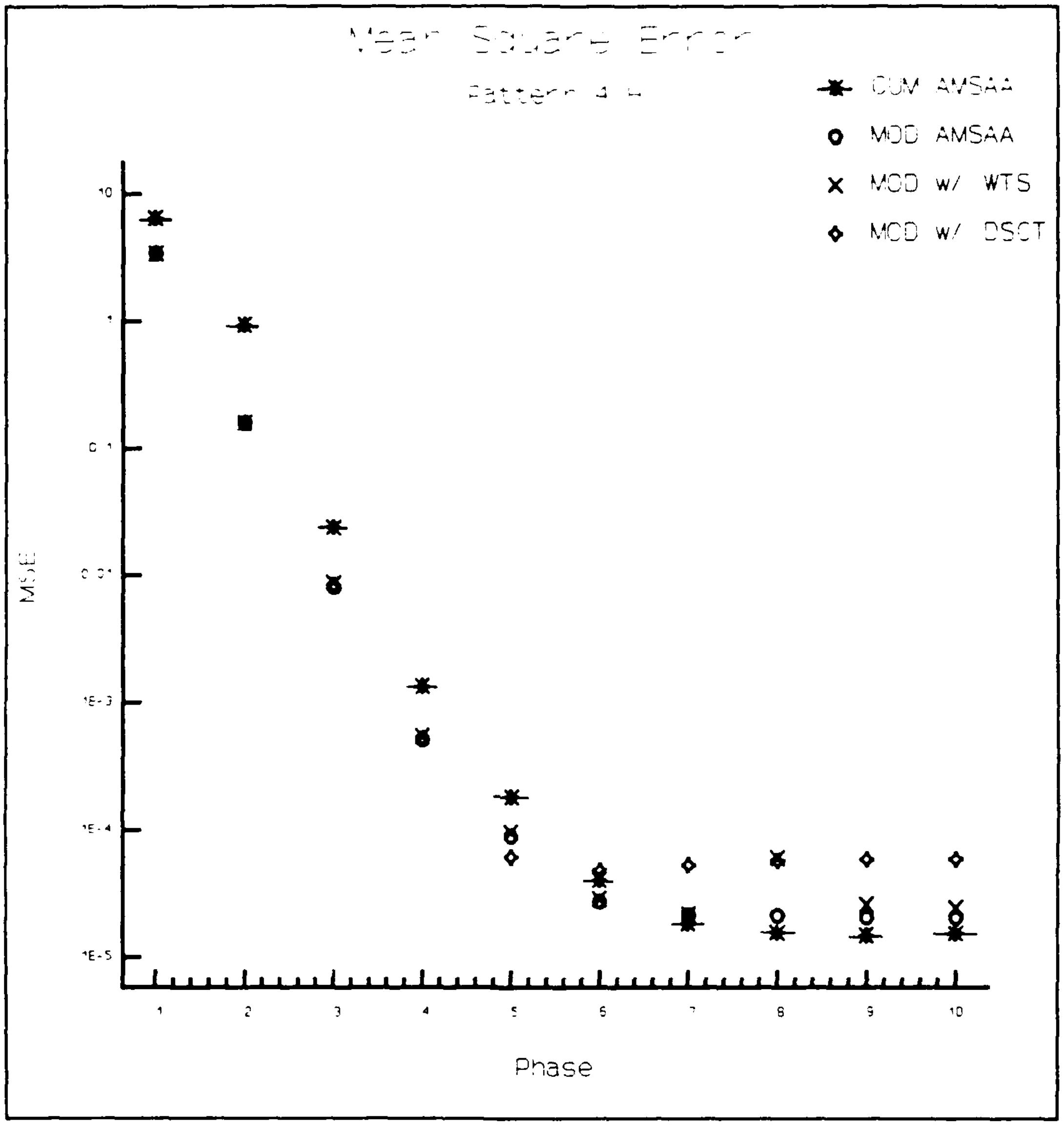
Avg. Est. Failure Rate
Pattern 3 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase
1
2
3
4
5
6
7
8
9
10



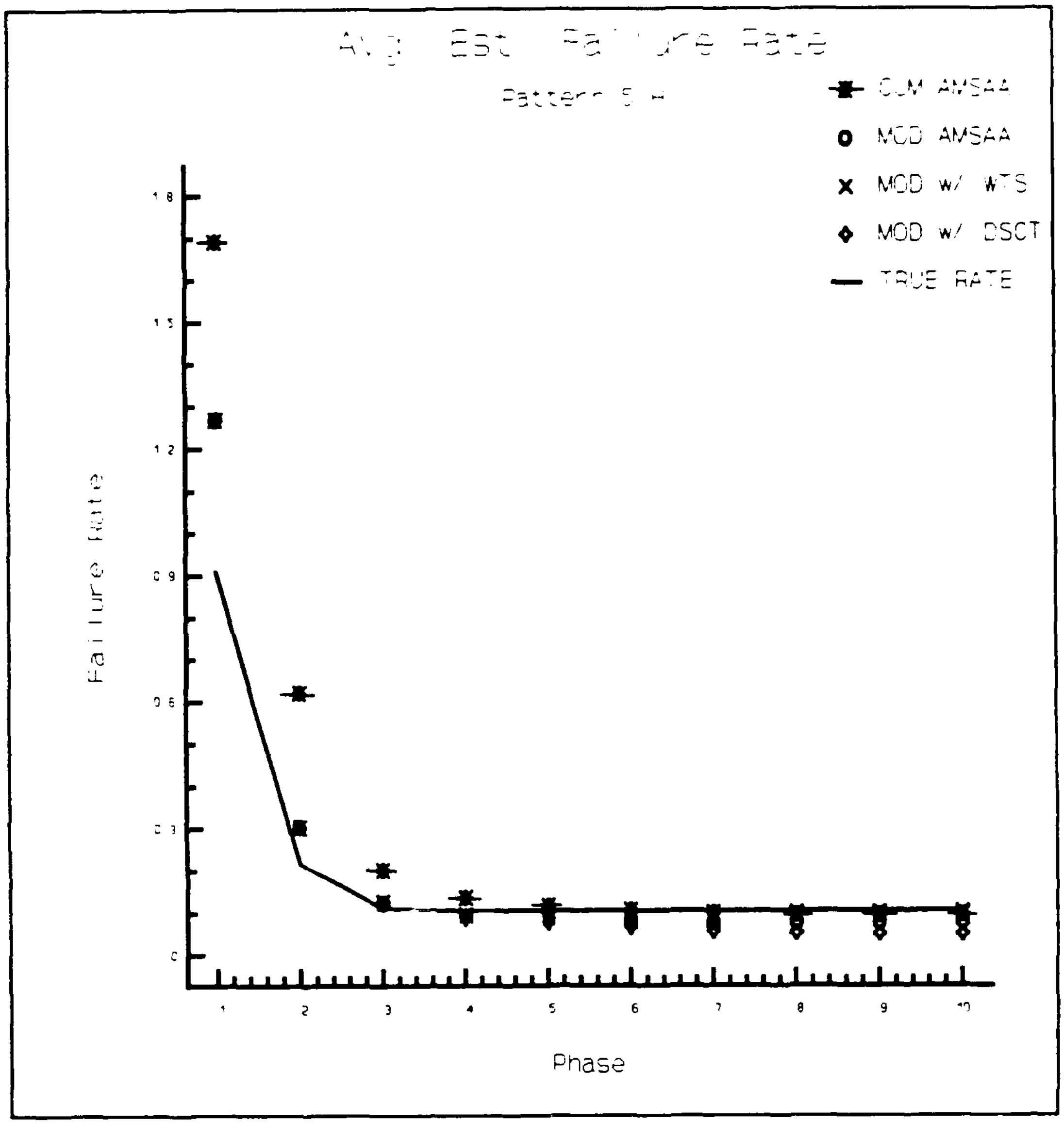
Mean Square Error
Pattern 3 -
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
10
1
0.1
0.01
1E-3
1
2
3
4
5
6
7
8
9
10
Phase



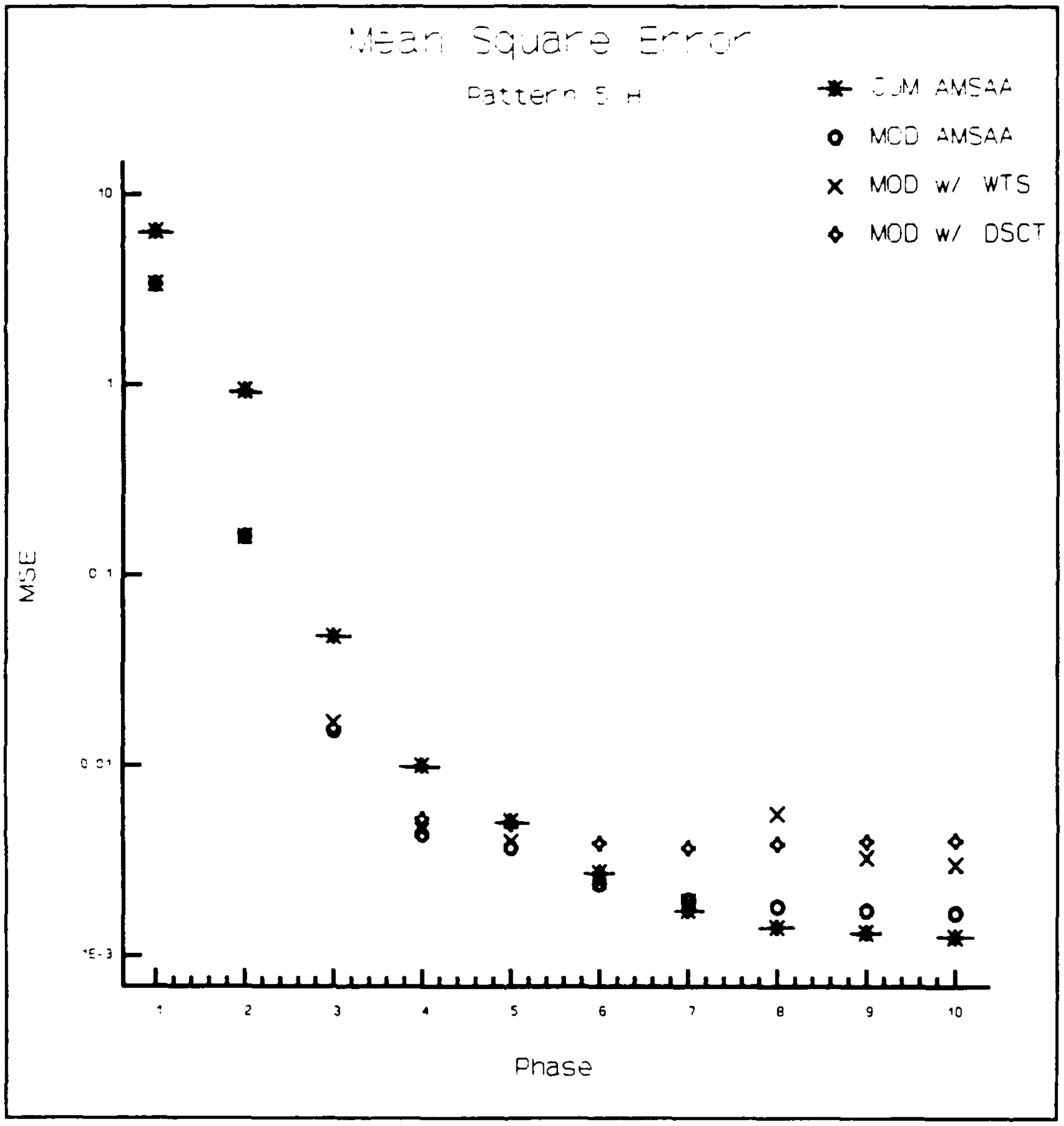
Avg. Est. Failure Rate
Pattern 4 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
1.8
1.5
1.2
0.9
0.6
0.3
0
1
2
3
4
5
6
7
8
9
10
Phase

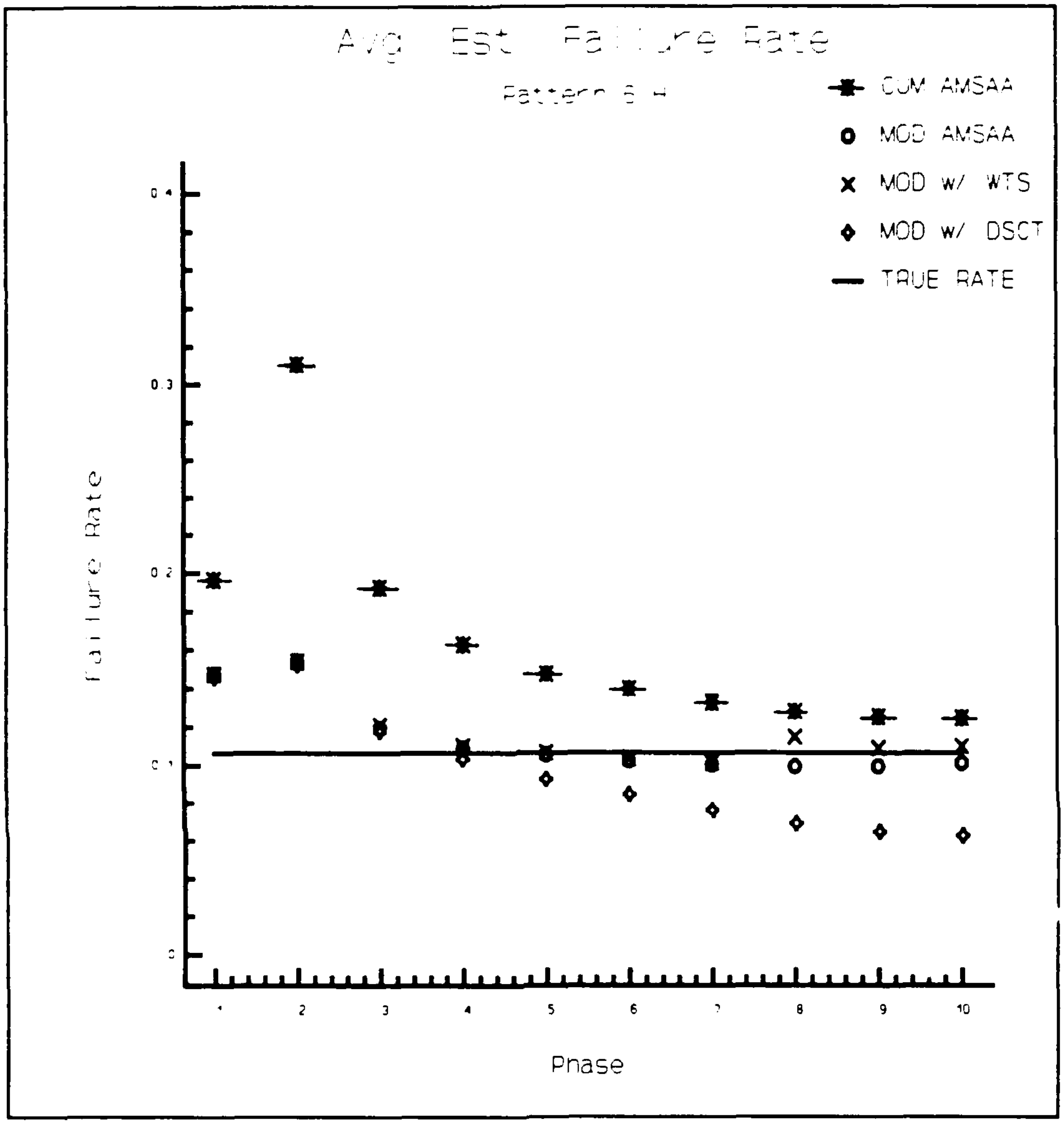


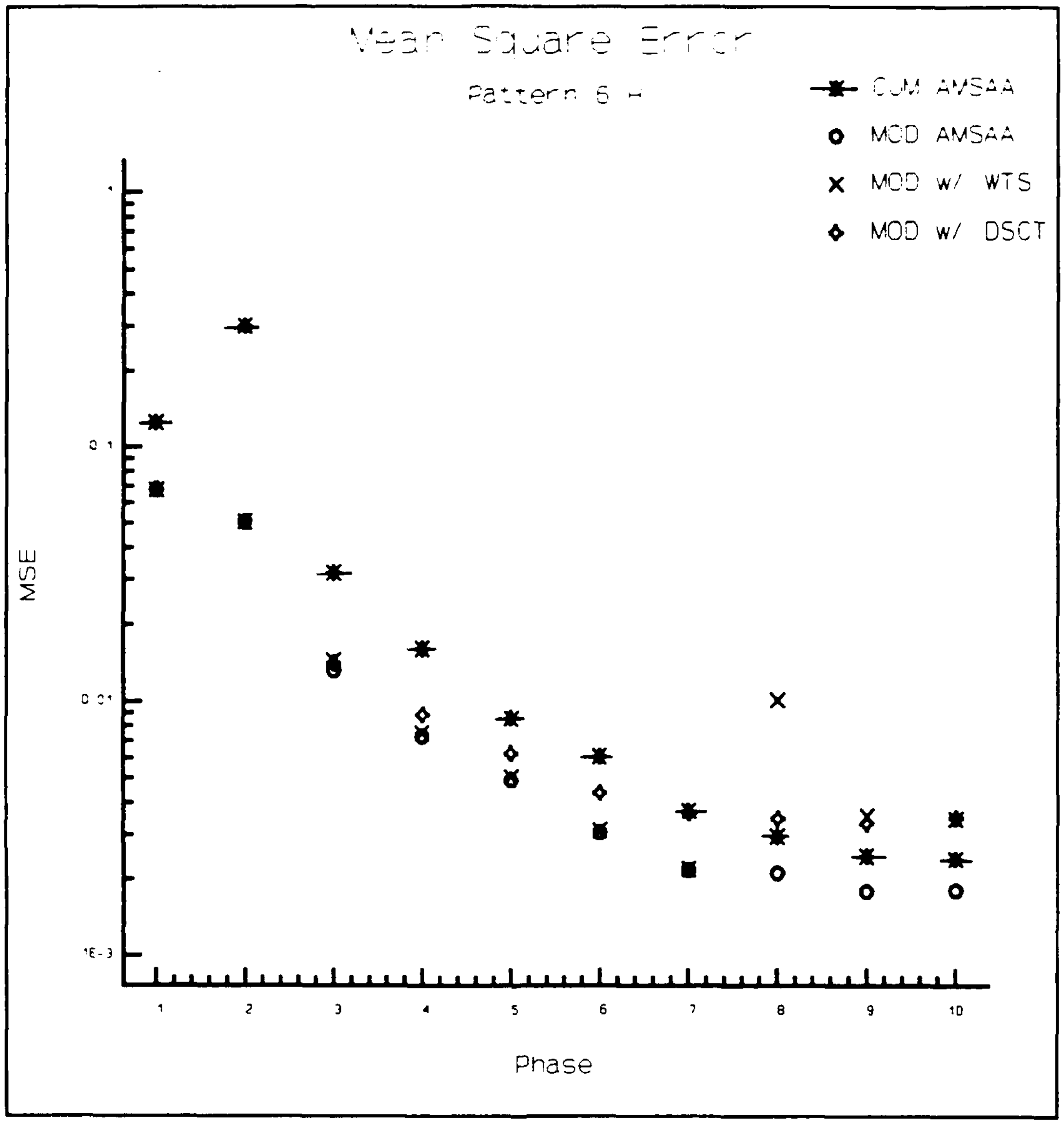
Pattern 4.4
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
10
1
1E-3
1E-4
1E-5
MSE
1
2
3
4
6
8
9
10
Phase

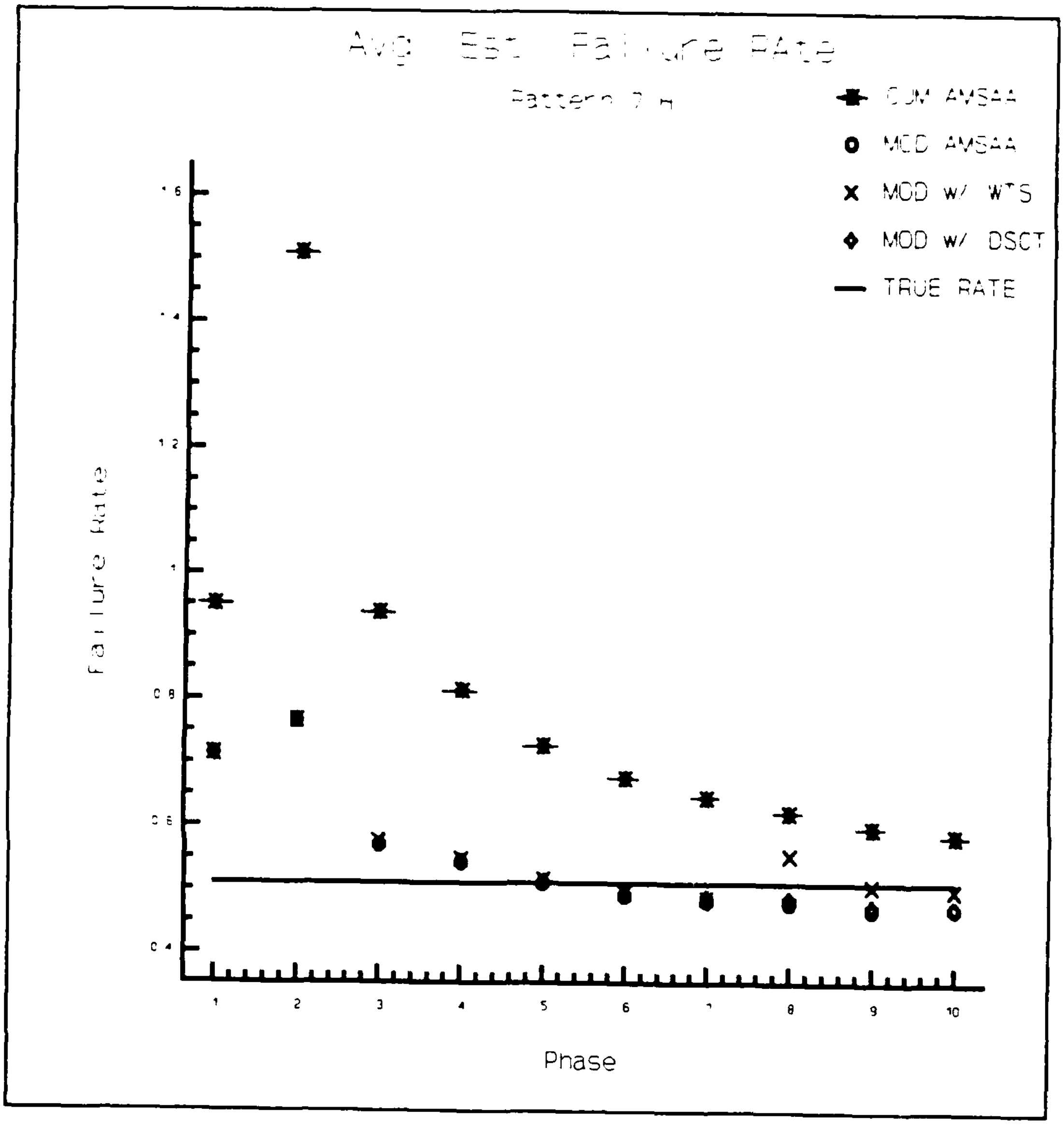


MOD AMSAA
MOD w/ DSCT
Failure Rate
1.8
1.5
1.2
0.9
0.6
0.3
0
1
2
3
4
5
6
7
8
9
Phase

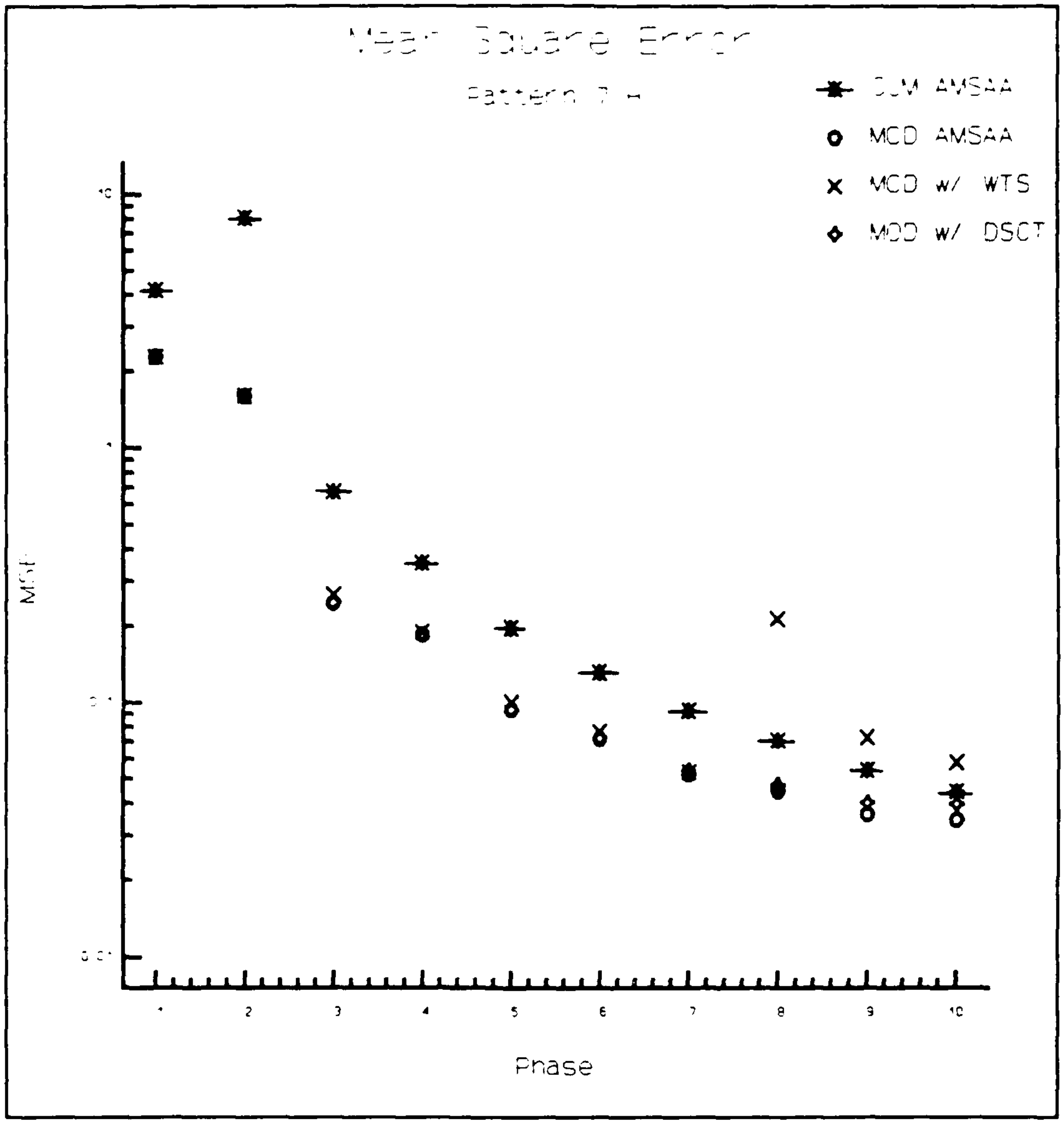

Mean Square Error
Pattern 5 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
10
1
0.1
0.01
1E-3
1
2
3
4
5
6
7
8
9
10
Phase

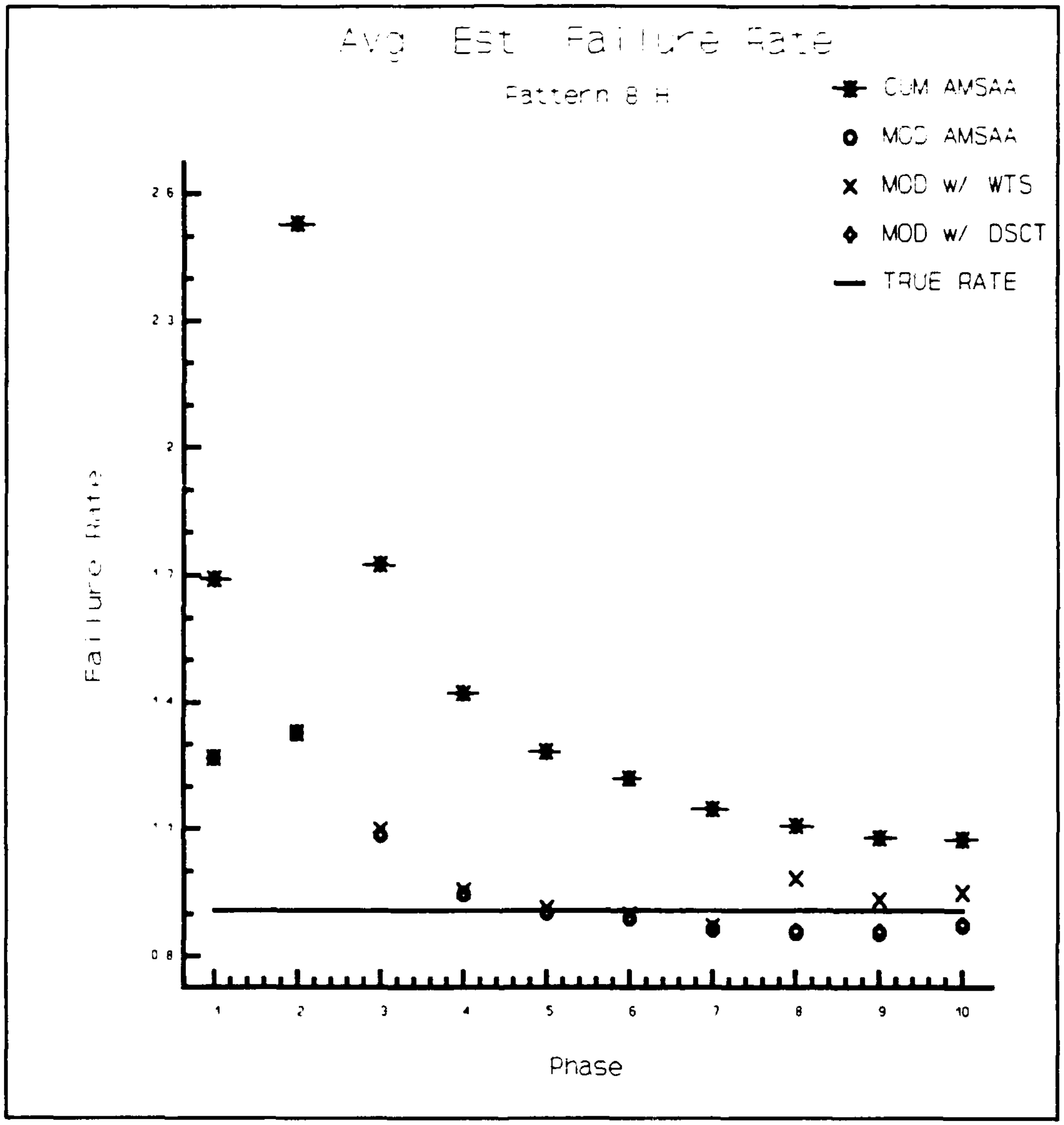

Avg Est Failure Rate
Pattern 6 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10
Phase


Mean Square Error
Pattern 6 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
MSE
0.1
0.01
1E-3
1
2
3
4
5
6
7
8
9
10
Phase

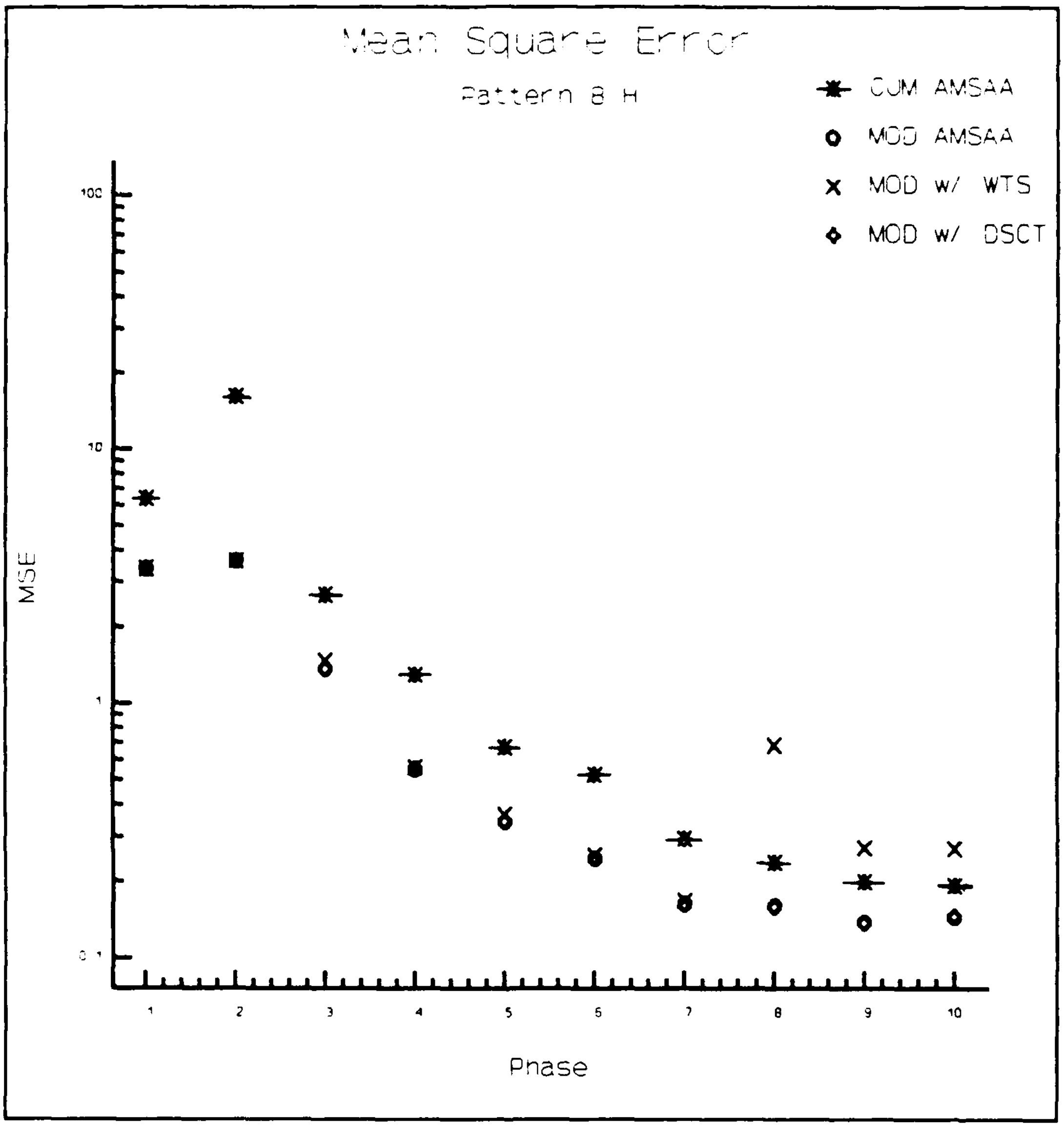


Avg. Est. Failure Rate
Pattern 7 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
1.6
1.4
1.2
1
0.8
0.6
0.4
1
2
3
4
5
6
8
9
10
Phase


MCD AMSAA
MCD w/ WTS
2
3
4
5
6
7
8
9
10
Phase



Avg Est Failure Rate
Pattern 8 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
TRUE RATE
Failure Rate
Phase


Mean Square Error
Pattern 8 H
CUM AMSAA
MOD AMSAA
MOD w/ WTS
MOD w/ DSCT
100
10
1
0.1
MSE
1
2
3
4
5
6
7
8
9
10
Phase

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